A Concise Analysis on Fuzzy Graph Theoretical Domination-I

R. Manikandan[1], S.Sivamani[2],

[1]Department of Mathematics, M.I.E.T. Engineering College, Trichy, India, manimaths78@gmail.com

[2]Department of Mathematics, Saranathan College of Engineering, Trichy, India,

winmayi2012@gmail.com

Abstract - In the fuzzy graph theory the rapid developing area is a study of domination and its related parameters. One of the important reason for this development is the domination in fuzzy graph theory plays a important role in several applications in engineering, social sciences, biology, computer science and some other fields. This motivates us to present this article. Here we list the few existing domination parameter. This will be a road map to the young researchers in this field.

Keywords: fuzzy graph,strong edge, dominating set,edge dominating set, connectedness, complement.

I. INTRODUCTION

From the inception of fuzzy mathematics by [1] L.A. Zadeh, it is considered as a large space research area. The concept of fuzzy graphs is introduced by [2] A. Rosenfeld and R.T. Yeh , S.Y. Bang in 1975. Since there are several applications for fuzzy graphs, several new concepts are defined in fuzzy graph theory. Particularly the domination concept was attracted by many researchers. In the last two decades several domination parameters are developed and also applied in many real time applications. So the development in this area will reduce the complexity, vagueness of technological problems. Fuzzy graph theory has numerous applications to problems in computer science, electrical engineering system analysis, operations research, economics, networking routing, transportation, etc.

II. EXISTING DOMINATION PARAMETERS IN FUZZY GRAPH

In this section few existing domination parameters in fuzzy graphs are illustrated.

The foundation of domination in fuzzy graphs was built by A. Somasundaram, S. Somasundaram in [3].They defined domination of a fuzzy graph $G = (V, E)$ as follows. A set of vertices $D \subseteq V$ is said to be a fuzzy dominating set of G if for every $v \in V - D$, there exists u in D such that $\mu(u, v) = \sigma(u) \wedge \sigma(v)$. The minimum scalar cardinality of D is called the fuzzy domination number and is denoted by $\gamma(G)$. This is the source for many domination parameters based on the conditions impost on either D or V-D. Also in that paper, independent and total domination parameters are defined. A dominating set D is called independent if $\mu(u, v) < \sigma(u) \wedge \sigma(v)$ for all u,v in D . If G is a fuzzy graph without isolated vertices then a dominating set is called total dominating if every vertex in V is dominated by a vertex in D.

The edge analogue of this domination parameter was originated by A. Nagoor Gani and K.Prasanna devi [4] by using strong arc. Let $G = (\sigma, \mu)$ be a fuzzy graph. Let e_i and e_j be two edges of G. We say that e_i dominates e_j if e_i is a strong arc in G and adjacent to e_j .A subset D of E(G) is said to be an edge dominating set of G if for every $e_j \in E(G) - D$, there exists $e_i \in D$ such that ei dominates e_j . The smallest number of edges in any edge dominating set of G is called its edge domination number and it is denoted by $\gamma'(G)$.

S.Ghobadi, D.Soner Nandappa and Q.M. Mahyoub[5] introducing a new domination for fuzzy graph by imposing an additional condition on V-D. Let D be the minimum dominating set of G. If V-D contains a dominating set D ' then D ' is called the inverse dominating set of G with respect to D. The inverse domination number is the minimum cardinality taken over all the minimal inverse dominating set of G denoted by $\gamma^{-1}(G)$.

C.Y.Ponnappan, S.Basheer Ahamed and P.Surulinathan[6] introducing the inverse edge domination for fuzzy. A subset D of X is a minimal edge dominating set of a fuzzy graph G. If V-D contains an edge dominating set D′ is called an inverse edge dominating set of G with respect to D. The minimum fuzzy edge cardinality taken over all inverse edge dominating set of G is called the inverse edge domination number of the fuzzy graph G and it is denoted by $\gamma_I'(G)$.

Based on the neighborhood degree of a vertex in a fuzzy graph C.Natarajan and S.K.Ayyaswamy [7] framed strong (weak) domination for fuzzy graph. If u and v are any two vertices of a fuzzy graph G, then u strongly dominates v (v weakly dominates u) if (i) $\mu(u, v) = \sigma(u) \wedge \sigma(v)$ and (ii)

 DOI : 10.18231/2454-9150.2018.0160

dN(u) ≥ dN(v). A vertex set D ⊆ V is said to be a strong (weak) fuzzy dominating set of G if every vertex v ∈ V –D is strongly (weakly) dominated by some vertex u in *D*. The strong (weak) fuzzy dominating set is denoted by sfd-set (wfd-set).The minimum scalar cardinality of a sfd-set (wfd-set) is called the strong (weak) fuzzy domination number of *G* and it is denoted by $\gamma_{sf}(G)$($\gamma_{wf}(G)$).

K.M. Dharmalingam , M. Rani[8] framed another domination based on the degree of the vertex. Let G be a fuzzy graph. Let u and v be two vertices of G. A subset D of V is called a fuzzy equitable dominating set if for every v ∈ V −D there exist a vertex u ∈ D such that uv ∈ E(G) and |deg(u) − deg(v)| ≤ 1 and μ(uv) ≤ σ(u)∧σ(v). The minimum cardinality of a fuzzy equitable dominating set is denoted by γ^{ef}.

It is clear that for domination, every element in V-D is related with at least one element in D. The condition that element in V-D is related with how many elements in D will be the reason for the following new domination parameters.

A subset D of V in a fuzzy graph G is a double dominating set of G if for each vertex in V is dominated by at least two vertices in D. The double domination number of a fuzzy graph G is the minimum fuzzy cardinality of a double dominating set D and is denoted by $\gamma_{dd}(G)$ which was developed by Q.M. Mahioub and N.D. Soner in [9].

A dominating set D of a fuzzy graph G is said to be a perfect dominating set if for each vertex v not in D, v is adjacent to exactly one vertex of D. A perfect dominating set D of a fuzzy graph G is said to be a minimal perfect dominating set if for each vertex v in D, D-{v} is not a dominating set. A perfect dominating set with smallest cardinality is called a minimum perfect dominating set. It is denoted by γ_{pf} set of G. The cardinality of a minimum perfect dominating set is called the perfect domination number of the fuzzy graph G. It is denoted by $\gamma_{pf}(G)$ which was developed by S.Revathi, P.J.Jayalakshmi and C.V.R. Harinarayanan in [10].

An edge dominating set X of a fuzzy graph G is said to be a perfect edge dominating set if every edge of E-X is adjacent to exactly one edge in X. The cardinality of a minimum perfect edge dominating set is called as perfect edge domination number and is denoted by $\gamma'_{pf}(G)$. This concept is developed by S.Ramya and S.Lavanya[11].

A vertex v in V-D is said to be fuzzy k-dominated if it is dominated by at least k vertices in D. In other words |N (v)∩ D|≤ k. In a fuzzy graph G every vertex in V-D is fuzzy k-dominated , then D is called a fuzzy k-dominating set. The minimum cardinality of a fuzzy k-dominating set is called the fuzzy k-domination number γk (G). A fuzzy dominating set D of a fuzzy graph G is called multiple dominating set of G if for each vertex in V-D be dominated by multiple(more than one vertex) vertices in D. This concept was initiated by G.Nirmala and M.Sheela in [12].

In a fuzzy graph G, the minimum cardinality set D of G which is the fuzzy dominating set of both G and $\overline{G}$ is called a fuzzy global dominating set. The minimum cardinality of a fuzzy global dominating set is called the fuzzy global domination number denoted by $\gamma_g(G)$.

A fuzzy graph H=(σ,μ) is said to have a fuzzy t-factoring into factors F(H)= {G_1,G_2,G_3,......G_t} if each fuzzy graph Gi=(σi,μi)such that σi=σ and the set{μ_1,μ_2,μ_3……..μ_t}form a fuzzy partition of μ. Given A t-factoring F of H, a subset D⊆V is a fuzzy factor dominating set if D is a fuzzy dominating set of Gi, for1≤i≤t.

The fuzzy factor domination number γ_{ft}(F(H)) is the minimum cardinality of a fuzzy factor dominating set of F(H). These parameters are also found by G.Nirmala and M.Sheela in [13].

The concept of perfect domination is generalized and extended that to edge analogue by S.Ramya and S.Lavanya[11].A vertex subset D of a fuzzy graph G is said to be a perfect k-vertex dominating set of G, if every vertex v of G not in D is adjacent to exactly k-vertices of D. The minimum cardinality of a perfect k-vertex dominating set of G is called the perfect k-domination number of the fuzzy graph G. It is denoted by $\gamma_{pf}(G)$.

An edge dominating set X of a fuzzy graph G is said to be a perfect k-edge dominating set if every edge of E-X is adjacent to exactly k edges in X. The cardinality of a minimum perfect k-edge dominating set is called as perfect k-edge domination number and is denoted by $\gamma'_{pf}(G)$.

In [14], Mohamed Ismayil, Ismail Mohideen enforcing another condition on V-D for a dominating set.

A set D ⊆ V is said to be a complementary nil dominating set (or simply called cnd-set) of a fuzzy graph G if

D is a dominating set and its complement V -D is not a dominating set. The minimum scalar cardinality taken

over all cnd-set is called a complementary nil domination number and is denoted by γ_{cnd}, the corresponding minimum cnd-set is denoted by γ_{cnd}-set.

The following parameters are based on the connectedness of D or V-D of a fuzzy graph G.

A dominating set *D* of a fuzzy graph *G* is a connected dominating set if the induced fuzzy subgraph $\langle D \rangle$ is connected . The minimum fuzzy cardinality of a connected dominating set of *G* is called the connected domination number of *G* and is denoted by $\gamma_c(G)$.

A dominating set D of a fuzzy graph G is a split dominating set if the induced fuzzy sub graph $\langle V - D \rangle$ is disconnected. The split domination number

 DOI : 10.18231/2454-9150.2018.0160

$\gamma_s(G)$ of G is the minimum fuzzy cardinality of a split dominating set. These were introduced by Q.M. Mahyoub and D.Soner Nandappa[15].

A dominating set D of G is said to be an fuzzy bridge independent dominating set, if ⟨D⟩ contains no bridge of G. The minimum cardinality of a fuzzy bridge independent dominating set of G is said to be the fuzzy bridge independent domination number and is denoted by $\gamma^{f}_{bi}(G)$. It was introduced by K. M. Dharmalingam and P. Nithya [16]

The following parameters are defined by A. Nagoor Gani and A. Arif Rahman in [17]. A dominating set D is said to be an accurate dominating set of a fuzzy graph G, if V−D has no dominating set of cardinality $|D|$. The accurate domination number of a fuzzy graph G, is denoted by (G), is the minimum cardinality taken over all accurate dominating sets of a fuzzy graph G.

Let G be a connected fuzzy graph and an accurate dominating set D, where D is a subset of V, of a fuzzy graph G is said to be a connected accurate dominating set if a fuzzy induced sub graph of ⟨D⟩ is connected. The connected accurate domination number of a fuzzy graph G is the minimum cardinality taken over all connected accurate dominating sets of a fuzzy graph G, and it is denoted by (G).

In their paper, S.Vimala and J.S.Sathya [18] defined a new parameter. A dominating set D of a fuzzy graph is said to be a point set dominating set of a fuzzy graph if for every $S \subseteq V-D$, there exists a node d∈D such that $\langle S-\{d\}\rangle$ is a connected fuzzy graph. The minimum cardinality taken over all minimal point set dominating set is called a point set domination number of a fuzzy graph G and it is denoted by $\gamma_p(G)$.

III. Conclusion

Graph theory is the one of the important branch of mathematics from its introduction. There are several parameters developed in graph theory like domination, labelling, colouring, etc. Fuzzy mathematics is attracted by many researchers in last two decades. It introduced several branches as in pure mathematics. One such area is fuzzy graph theory. Researchers developed many parameters in it like domination. The different domination parameters are defined based on the specific condition. In this paper several domination parameters are referred to increase the interest on domination in fuzzy graph. Also hope, it motivates the young researchers to find out new domination parameters suitable for real time problems.

Acknowledgment

The authors are grateful to the referee for their valuable comments and suggestions for improving the paper.

References

[1] A.Somasundaram, and S.Somasundaram, "Domination in fuzzy graphs-I", Pattern Recognition Letters, 19, 1988, 787-791.

[2] A. Nagoorgani and K.Prasanna Devi, "Edge domination and independence in fuzzy graph", Advances in fuzzy sets and systems, Volume 1, Issue1,2006,17-26.

[3]S.Ghobadi, D.Soner Nandappa and Q.M. Mahyoub , "*Inverse dominating set in fuzzy graphs*",. Proc. Jangjeon Math. Soc., 11 (1), 2008, 75-81.

[4]C.Y.Ponnappan, S.Basheer Ahamed and P.Surulinathan, "Inverse Edge Domination in Fuzzy Graphs", International Journal of Mathematics Trends and Technology, Volume 20 , Number 2, April 2015, 103-109.

[5]C.Natarajan.C and S.Ayyaswamy, "Strong (weak) domination in fuzzy graphs", International Journal of Computational and Mathematical sciences, 2010, 526-528.

[6]K.M. Dharmalingam , M. Rani, " Equitable domination in fuzzy graphs", International Journal of Pure and Applied Mathematics, Volume 94 No. 5 2014, 661-667.

[7]Q.M. Mahioub and N.D. Soner, " The double domination number of fuzzy graphs" , Journal of Applied Mathematical Analysis and Applications, 3(2) ,2007, 97-104.

[8]S.Revathi,P.J.Jeyalakshmi,C.V.R.Harinarayanan, "Perfect dominating sets in fuzzy graphs", IOSR journal of mathematics,Volume-8 , Issue-3,43-47,2013.

[9]S.Ramya and S.Lavanya, " Perfect Vertex (Edge) Domination in Fuzzy Graphs" , Indian Journal of Science and Technology, Vol 9(4),2016,1-5.

[10]G.Nirmala , M.Sheela, " Fuzzy Multiple Domination", International Journal of Scientific and Research Publications, Volume 3, Issue 12, December 2013,1-3.

[11]G.Nirmala , M.Sheela, "Global and Factor Domination in Fuzzy Graph", International Journal of scientific and Research Publications, Volume2,Issue6, June 2012, 1-4.

[12]Mohamed Ismayil, Ismail Mohideen, Complementary nil domination in fuzzy graphs, Annals of fuzzy mathematics and informatics, Vol.X,No X, 2014, 1-8.

[13]Q.M.Mahyoub, and N.D.Soner, "The split domination number of fuzzy graphs" Far East J. Appl. Math., Vol.30 (1), 2008, 125-132.

[14] K. M. Dharmalingam and P. Nithya , "Excellent domination in fuzzy graphs", Bulletin of the International Mathematical Virtual Institute, Vol. 7(2017), 257-266.

[15]A. Nagoor Gani and A. Arif Rahman, "Some Properties on Connected Accurate Domination in Fuzzy Graphs", Global Journal of Pure and Applied Mathematics, Volume 13, Number 9 2017, 4983-4993.

[16]S.Vimala and J.S.Sathya, "Some Results on Point Set Domination of Fuzzy Graphs" Cybernetics and Information Technologies , Volume 13, No 2,2013, 58-62.

 DOI : 10.18231/2454-9150.2018.0160