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(1,2)*- β g‴ -interior and (1,2)*- β g‴ -closure in Bitopological Spaces

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Abstract: The purpose of this paper for the notion of (1,2)*- β g‴ -interior is defined and some of its basic properties are studied. Also we introduce the concept of (1,2)*- β g‴ - closure in bitopological spaces using the notion of (1,2)*- β g‴ -closed sets, and we obtain some related results. For any A ⊆ X, it is proved that the complement of (1,2)*- β g‴ -interior of A is the (1,2)*- β g‴ -closure of the complement of A.

Keywords: (1,2)*- β g‴ -interior and closure, (1,2)*- β g‴ -closed set, (1,2)*- β g‴ -open set, (1,2)*-g̈-open.

I. INTRODUCTION

The first step of locally closedness was done by Bourbaki [1]. He defined a set A to be locally closed if it is the intersection of an open set and a closed set. In literature many general topologists introduced the studies of locally closed sets. Extensive research on locally closedness and generalizing locally closedness were done in recent years. Stone [2] used the term FG for a locally closed set. Ganster and Reilly used locally closed sets in [3] to define LC-continuity and LC-irresoluteness. Balachandran et al introduced the concept of generalized locally closed sets. Veera Kumar (Sheik John [4]) introduced ĝ-locally closed sets (= ω -locally closed sets) respectively.

II. PRELIMINARIES

Definition: 2.1 A subset A of a space (X, τ_1, τ_2) is called:

(i) a (1,2)*-ĝ-closed set [5] (= (1,2)*-ω-closed set) if $\tau_{1,2}$-cl(A) ⊆ U whenever A ⊆ U and U is (1,2)*-semi-open in (X, τ_1, τ_2). The complement of (1,2)*-ĝ-closed set is called (1,2)*-ĝ-open set;

(ii) a (1,2)*-g̈ -closed set [6] if $\tau_{1,2}$-cl(A) ⊆ U whenever A ⊆ U and U is (1,2)*-sg-open in (X, τ_1, τ_2). The complement of (1,2)*- g̈-closed set is called (1,2)*-g̈-open.

Remark: 2.2

(i) Every $\tau_{1,2}$-open set is (1,2)*- g‴ -open but not conversely.

(ii) Every $\tau_{1,2}$-closed set is (1,2)*- g‴ -closed but not conversely.

(iii) Every (1,2)*- g‴ -closed set is (1,2)*- g̈-closed but not conversely.

(iv) Every (1,2)*- g̈ -closed set is (1,2)*- ω -closed but not conversely.

(v) Every (1,2)*- g‴ -closed set is (1,2)*- ω -closed but not conversely.

Proposition: 2.3

If A and B are (1,2)*- g‴ -closed sets in (X, τ_1, τ_2), then A ∪ B need not be (1,2)*- g‴ -closed in (X, τ_1, τ_2).

III. (1,2)*- β g‴ -INTERIOR

Definition: 3.1

For any A ⊆ X, (1,2)*- β g‴ -int(A) is defined as the union of all (1,2)*- β g‴ -open sets contained in A. i.e., (1,2)*- β g‴ -int(A) = ∪ {G : G ⊆ A and G is (1,2)*- β g‴ -open} [7].

Lemma: 3.2

For any A ⊆ X, $\tau_{1,2}$-int(A) ⊆ (1,2)*- β g‴ -int(A) ⊆ A.

Proof:

It follows from Remark 2.2 (i).

The following two Propositions are easy consequences from definitions.

Proposition: 3.3

For any subsets A and B of (X, τ_1, τ_2),

(i) (1,2)*- β g‴ -int(A ∩ B) = (1,2)*- β g‴ -int(A) ∩ (1,2)*- β g‴ -int(B).

(ii) (1,2)*- β g‴ -int(A ∪ B) ⊇ (1,2)*- β g‴ -int(A) ∪ (1,2)*- β g‴ -int(B).

(iii) If A ⊆ B, then (1,2)*- β g‴ -int(A) ⊆ (1,2)*- β g‴ -int(B).

(iv) (1,2)*- β g‴ -int(X) = X and (1,2)*- β g‴ -int(φ) = φ.

IV. (1,2)*- β g‴ -CLOSURE

In this section, we define (1,2)*- β g‴ -closure of a set and study its properties

Definition : 4.1

For every set A ⊆ X, we define the (1,2)*- β g‴ -closure of A to be the intersection of all (1,2)*- β g‴ -closed sets containing A.

In symbols, (1,2)*- β g‴ -cl(A) = ∩ {F : A ⊆ F ∈ (1,2)*- G″ C(X)}.

Lemma: 4.2

For any A ⊆ X, A ⊆ (1,2)*- β g‴ -cl(A) ⊆ $\tau_{1,2}$-cl(A).

Proof: It follows from Remark 2.2 (ii).

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Remark: 4.3

Both containment relations in Lemma 4.2 may be proper as seen from the following example.

Example : 4.4

Let X = {a, b, c}with τ1 = {φ, {a, b}, X} and τ_2 = {φ, X}. Let A = {a}. Then (1,2)*- β g‴ -cl(A) = {a, c} and so A ⊂ (1,2)*- β g‴ -cl(A) ⊂ $\tau_{1,2}$-cl(A).

Lemma: 4.5

For any A ⊆ X, (1,2)*- ω -cl(A) ⊆ (1,2)*- β g‴ -cl(A), where (1,2)*- ω - cl(A) is given by (1,2)*-ω-cl(A) = ∩ {F : A ⊆ F ∈ (1,2)*- ω C(X)}.

Proof:

It follows from Remark 2.2 (v).

Containment relation in the above Lemma 4.5 may be proper as seen from the following example.

Example: 4.6

Let X = {a, b, c, d}with τ1 = {φ, {a}, {b, c}, {a, b, c}, X} and τ_2 = {φ, X, {a}}. Then (1,2)*- G″ C(X) = {φ, {d}, {a, d}, {b, c, d}, X} and (1,2)*- ω C(X) = {φ, {d}, {a, d}, {b, d}, {c, d}, {a, b, d}, {a, c, d}, {b, c, d}, X}.

Let A= {b, d}. Then (1,2)*- β g‴ -cl(A) = {b, c, d} and (1,2)*- ω -cl(A) = {b, d}. So, (1,2)*-ω-cl(A) ⊂ (1,2)*- β g‴ -cl(A) [8].

Lemma: 4.7

For an x ∈ X, x ∈ (1,2)*- β g‴ -cl(A) if and only if V ∩ A ≠ φ for every (1,2)*- β g‴ -open set V containing x.

Proof:

Let x ∈ (1,2)*- β g‴ -cl(A) for any x ∈ X. To prove V ∩ A ≠ φ for every (1,2)*- β g‴ -open set V containing x. Prove the result by contradiction. Suppose there exists a (1,2)*- β g‴ -open set V containing x such that V ∩ A = φ. Then A ⊂ V^c and V^c is (1,2)*- β g‴ -closed. We have (1,2)*- β g‴ -cl(A) ⊂ V^c. This shows that x ∉ (1,2)*- β g‴ -cl(A) which is a contradiction. Hence V ∩ A ≠ φ for every (1,2)*- β g‴ -open set V containing x.

Conversely, let V ∩ A ≠ φ for every (1,2)*- β g‴ -open set V containing x. To prove x ∈ (1,2)*- β g‴ -cl(A). We prove the result by contradiction. Suppose x ∉ (1,2)*- β g‴ -cl(A). Then there exists a (1,2)*- β g‴ -closed set F containing A such that x ∉ F. Then x ∈ F^c and F^c is (1,2)*- β g‴ -open. Also F^c ∩ A = φ, which is a contradiction to the hypothesis. Hence x ∈ (1,2)*- β g‴ -cl(A) [9].

Proposition 4.8

For any two subsets A and B of (X, τ_1, τ_2), the following hold:

(i) If A ⊆ B, then (1,2)*- β g‴ -cl(A) ⊆ (1,2)*- β g‴ cl(B).

(ii) (1,2)*- β g‴ -cl(A ∩ B) ⊆ (1,2)*- β g‴ -cl(A) ∩ (1,2)*- β g‴ -cl(B).

Theorem: 4.9

Let A be any subset of X. Then

(i) ((1,2)*- β g‴ -int(A))^c = (1,2)*- β g‴ -cl(A^c).

(ii) (1,2)*- β g‴ -int(A) = ((1,2)*- β g‴ -cl(A^c))^c.

(iii) (1,2)*- β g‴ -cl(A) = ((1,2)*- β g‴ -int(A^c))^c.

Proof:

(i)Let x ∈ ((1,2)*- β g‴ -int(A))^c. Then x ∉ (1,2)*- β g‴ -int(A). That is, every (1,2)*- β g‴ -open set U containing x is such that U ⊄ A. That is, every (1,2)*- β g‴ -open set U containing x is such that U ∩ A^c ≠ φ. By Lemma 4.8 x ∈(1,2)*- β g‴ -cl(Ac) and therefore ((1,2)*- β g‴ -int(A))^c ⊆ (1,2)*- β g‴ -cl(A^c). Conversely, let x ∈ (1,2)*- β g‴ -cl(A^c). Then by Lemma 4.8 every (1,2)*- β g‴ -open set U containing x is such that U ∩ A^c ≠ φ. That is, every (1,2)*- β g‴ -open set U containing x is such that U ⊄ A. This implies by Definition 3.1 x ∉ (1,2)*- β g‴ -int(A). That is, x ∈ ((1,2)*- β g‴ -int(A))^c and so (1,2)*- β g‴ -cl(A^c) ⊆ ((1,2)*- β g‴ -int(A))^c. Thus ((1,2)*- β g‴ -int(A))^c = (1,2)*- β g‴ -cl(A^c).

(ii) Follows by taking complements in (i).

(iii) Follows by replacing A by A^c in (i) [10].

REFERENCE

[1] Bourbaki, N.: General topology, Part I, Addison-Wesley, Reading, Mass., 1966.

[2] Stone, M.: Application of the theory of Boolean rings to general topology, Trans. Amer. Math. Soc., 41 (1937), 374-481.

[3] Ganster, M. and Reilly, I. L.: Locally closed sets and LC-continuous functions, Internat J. Math. Sci., 12(3) (1989), 417-424.

[4] Sheik John, M.: A study on generalizations of closed sets and continuous maps in topological and bitopological spaces, Ph.D Thesis, Bharathiar University, Coimbatore, September 2002

[5] Ravi, O., Thivagar, M. L. and Jinjinli.: Remarks on extensions of (1,2)*-g-closed maps, Archimedes J. Math., 1(2) (2011), 177-187.

[6] Ravi, O., Kamaraj, M., Pandi, A. and Kumaresan, K.: (1,2)*-$\ddot{g}$ -closed and (1,2)*- $\ddot{g}$ -open maps in bitopological spaces, International Journal of Mathematical Archive, 3(2) (2012), 586-594.

[7] Ravi, O., Pandi, A. and Latha, R.: Contra-pre-(1,2)*-semi-continuous functions, Bessel Journal of Mathematics (To appear).

[8] Ravi, O., Pious Missier, S. and Salai Parkunan, T.: On bitopological (1,2)*-generalized homeomorphisms, Int J. Contemp. Math. Sciences.,5(11) (2010), 543-557.

[9] Ravi, O., Thivagar, M. L. and Nagarajan, A.: (1,2)*- α g-closed sets and (1,2)*-g α -closed sets (submitted).

[10] Ravi, O., Thivagar, M. L. and Ekici, E.: Decomposition of (1,2)*- continuity and complete (1,2)*-continuity in bitopological spaces, Analele Universitatii Din Oradea. Fasc. Matematica Tom XV (2008),29-37.

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