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Vague A Generalized Homeomorphism in Vague Topological Spaces

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Abstract : **In this we introduce vague α generalized closed and open mappings, vague α generalized homeomorphism, vague M α generalized homeomorphism, vague contra α generalized continuous mappings and also we derive the relation between various vague contra closed sets.**

Keywords — Vague α generalized , Vague α generalized homeomorphism, Vague contra α generalized continuous, vague M α generalized homeomorphism, Vague contra α generalized continuous.

I. INTRODUCTION

Levine[9] started the study of generalized closed sets in topological spaces in 1970.The concept of fuzzy sets was introduced by Zadeh [11] in 1965.Fuzzy topology was introduced by C.LChang[1] in1967. H.Maki ,k.balachandran ,R.Devi [6] introduced the concept of alpha generalized closed sets in topological spaces. The notion of contra continuous was introduced and investigated by Dontchev[3]. After that gau and buhrer[4] introduced the concept of vague sets .

In this paper we introduced the concept vague α generalized continuous mapping ,vague contra α generalized continuous mapping, vague α generalized homeomorphism and vague M α generalized homeomorphism and also we derive some relationships of vague α generalized continuous mappings with other vague generalized closed mappings ,Vague contra continuity ,Vague contra α generalized continuity and then we derive the relationships of vague contra α generalized closed sets with other generalized closed sets.

II. PRELIMINARIES

A vague set V in the universe of discourse P is characterized by two membership functions given by, a true membership function t_V: P → [0,1] and a false membership function f_V:P → [0,1].The grade of membership of x in the vague set V is bounded by a sub interval $[t_V(x),1-f_V(x)]$ of [0,1].This indicates that ,if the actual grade of membership μ(x),then $t_V(x) \leq \mu(x) \leq 1-f_V(x)$.The vague set V is written as $V=\{x,<[t_V(x),1-f_V(x)]>/x \in P\}$,where the interval $[t_V(x),1-f_V(x)]$ is called the vague value of x in V and is denoted by $V_V(x)$.

Definition2.1[10]: Let A and B be the vague sets of the form $A=\{z,<[t_A(z),1-f_A(z)]\ /z \in P\}$ and $B=\{z,<[t_B(z),1-f_B(z)]\ /z \in P\}$where P is a universal set , then

1.$A \subseteq B$ iff $t_A(z) \leq t_B(z)$ and $1-f_A(z) \leq 1-f_B(z)$ $\forall z \in P$.

2.$A^C ==\{z,<[f_A(z),1-t_A(z)]\ /z \in P\}$

3.$A \bigcap B==\{z,<[t_A(z) \wedge t_B(z),1-f_A(z) \wedge 1-f_B(z)]\ /z \in P\}$

4.$AUB==\{z,<[t_A(z) \vee t_B(z),1-f_A(z) \vee 1-f_B(z)]\ /z \in P\}$.

Definition 2.2[10]: A Vague topology on X is a family σ of vague sets in X satisfying the following axioms .

1. $0,1 \in \sigma$
2. $G_1 \cap G_2 \in \sigma$ for $G_1, G_2 \in \sigma$.
3. $\bigcup G_{i,i \in I} \in \sigma$, for any family $G_i \in \sigma$.

In this paper (X, σ) is called a vague topological space and any vague set in σ is known as a vague open set in X.

Definition 2.3[10]: Let (X ,τ) be a vague topological space and $A=\{x,<[t_A(x),1-f_A(x)]\}$ be a vague set in X. Then the vague interior and vague closure are defined by Vint(A)=U{G/G is a vague open set in X and G is contained in A}, Vcl(A)=∩ {G/G is a vague closed sets in X and G is containing A}.

Definition 2.4[10]:- Let (X ,τ) be a vague topological space. A vague set $A=\{x,<[t_A(x), 1-f_A(x)]\}$ of X is called

1. a vague closed set vcl(A)=A. The complement of a vague closed set is vague open set.
2. a vague α closed(VαC) set Vcl(Vint(Vcl(A))) $\subseteq$ A.

Remark :- The complement of above closed sets are open.

Definition 2.5[10]: A vague set $A=\{x,<[t_A(x),1-f_A(x)]\}$ of a vague topological space (X ,τ) is said to be a vague generalized closed set(VGCS) if Vcl(A)$\subseteq$ U whenever A $\subseteq$

U and U is a vague open set in X, and A^c is a vague generalized open set.

Definition 2.6[10]: A vague set A of a vague topological space (X ,τ) is said to be a vague generalized closed set(VαGCS) if Vαcl(A)$\subseteq$ U whenever A$\subseteq$ U and U is a vague open set in X and A^c is a vague α generalized open set.

Definition 2.7[8]: Let f:(X ,τ) →(y ,σ) is a bijective mapping where (X ,τ) and (y ,σ) are topological spaces, then f is said to be a homeomorphism if f and f^{-1} are continuous mappings.

Definition 2.8[3]: A function f:-X→ Y is called contra continuous if the inverse image of each open set in Y is a closed set in X.

III. VAGUE α GENERALISED CLOSED AND OPEN MAPPINGS

Definition 3.1 : A map g :(P ,τ) →(Q ,σ) is called a vague α generalized closed mapping if g(F) is a vague α generalized closed set in Q for each vague closed set 'F' in 'P'.

Example3.2: Let P ={a,b} and Q ={c,d} and G_1={<a,[0.4,0.6] >,< b,[0.5,0.6]>},G_2={< a,[0.7,0.8] >,<b ,[0.5,0.7] >}, G_3={<a,[0.3,0.8] >,< b,[0.5,0.7]>} .Then τ ={0,G_1,G_2,1} and σ={0,G_3,1} are vague topological spaces on P and Q respectively. Construct a map g: (P,τ) → (Q ,σ) by g(a)= a^c =c , g(b)=b^c=d.

Let A={< a,[0.4,0.6] >,<b, [0.4,0.5]>} is a vague closed set in (P,τ) then g(A) = {<a,[0.4,0.6] >,<b ,[0.5,0.6]>} is a vague α generalized closed set in (Q ,σ) .

Definition 3.3 : A map g: (P ,τ) →(Q ,σ) is called a vague α generalized open mapping if g(A) is a vague α generalized open set in Q for each vague open set 'A' in 'P'.

Definition 3.4 : A map g :(P ,τ) →(Q ,σ) is called a vague M α generalized closed mapping if g(A) is a vague α generalized closed set in Q for each vague α generalized closed set 'A' in 'P'.

Example3.5: Let P= {a ,b} and Q ={c ,d} and V_1={<a,[0.5,0.7] >,< b,[0.3,0.5]>},V_2={<a,[0.4,0.6]>,< b,[0.6,0.7]>}. Then τ={0,V_1,1} and σ={0,V_2,1} are vague topological spaces on P and Q respectively. Define a mapping g :(P ,τ) → (Q ,σ) by g(a)=b=c, g(b)=a=d. Let A={ <a, [0.3,0.4]>,< b ,[0.4,0.6]>} is a vague α generalized closed set in P. So g(A) = { <c,[0.4,0.6] >,< d ,[0.3,0.4]>} is also a vague α generalized closed set in Q.

Definition 3.6: A map g : (P ,τ) →(Q ,σ) is called a vague M α generalized open mapping if g(V) is a vague α generalized open set in Q for each vague α generalized open set 'V' in 'P'.

Definition 3.7: If (P ,τ) is a vague topological space and A={x,<t_A(x),1-f_A(x)>} is a vague set in P. Then Vαg closure of A (Vαgcl(A)) is defined as Vαgcl(A)= ∩ {G/G is a vague αg closed set A $\subseteq$G} and Vαg interior of A (Vαgint(A)) is defined as Vαgint(A)= U{G/G is a vague αg open set G $\subseteq$A}

Theorem 3.8: Every vague closed mapping is a vague α generalized closed map but not conversely.

proof: If g : (P ,τ) →(Q ,σ) is a vague closed mapping .Let A be a vague closed set in P, g(A) is a vague closed set in Q. Since every vague closed set is a vague α generalized closed set . so g(A) is a vague α generalized closed set in Q. So g is a vague α generalized closed map.

Converse need not be true , it seen from the following example,

Example3.9: Let P = {a ,b} and Q={c ,d} and G_1={<a ,[0.5,0.7] >,< b,[0.3,0.5]>},G_2={<a,[0.4,0.6] >,< b,[0.6,0.7]>}.Then τ={0,G_1,1} and σ={0,G_2,1} are vague topological spaces on P and Q respectively. Define a mapping g :(P ,τ) → (Q ,σ) by g(a)=b=c ,g(b)=a=d, Let A={<a, [0.3,0.5] >,< b,[0.5,0.7]>} is a vague closed set. Then g(A)= ={<a, [0.5,0.7] >,< b,[0.3,0.5]>} is a vague α generalised closed set. But g(A) is not a vague closed set.

Theorem 3.10: Every vague α closed map is a vague α generalized closed map but not converse.

proof : If f:(P ,τ) →(Q ,σ) is a vague α closed mapping .Let A be a vague closed set in P. f(A) is a vague closed set in Q. Since every vague α closed set is a vague α generalized closed set .So f(A) is a vague α generalized closed set in Q. So f is a vague α closed map.

Converse may not be true , it seen from the following example,

Example3.11: Let P = {a ,b} and Q={c ,d} and G_1={<x,[0.5,0.7],[0.3,0.5]>},G_2={<x,[0.4,0.6],[0.6,0.7]>}.Then τ ={0,G_1,1} and σ ={0,G_2,1} are vague topological spaces on P and Q respectively. Define a mapping g: (P,τ) → (Q,σ) by g(a)=b=d ,g(b)=a=c. Let A={ <[0.3,0.5],[0.5,0.7]>} is a vague closed set in P. So g(A)= { <[0.5,0.7],[0.3,0.5]>} is a vague α generalized closed set but not vague α closed set in Q.

Theorem 3.12: Every vague M α generalized closed map is a vague α generalized closed map.

proof : If g: (P ,τ) →(Q ,σ) is a vague M α generalized closed mapping .Let F be a vague closed set in Q. F is a vague α generalized closed set in Q. Since g is a vague M α generalized closed map, g(F) is a vague α generalized closed set in Q. So g is a vague α generalized closed map.

Theorem 3.13: If g: (P ,τ) →(Q ,σ) is a vague α generalized closed mapping and A is a vague closed set in P then g_A: A→ Q is a vague α generalized closed map.

proof : Let B $\subseteq$A is a vague α generalized closed set in P then B is a vague α generalized closed set in P. Since g is a vague α generalized closed map. g(B) is a vague α generalized closed set in Q. So g_A: A → Q is a vague α generalized closed map.

Definition 3.14: A map g: (P ,τ) →(Q ,σ) is called a vague α generalized irresolute map if $g^{-1}(V)$ is a vague α generalized closed set in P, for every vague α generalized closed set V in Q.

Theorem 3.15: If f:(P ,τ) →(Q ,σ) is a vague α generalized map and g: (Q ,σ) → (R, γ) are two vague M α generalized maps then gof: (P ,τ) →(R, γ) is a vague α generalized closed map.

proof : Let V be a vague α generalized closed set in (P ,τ). This implies f(V) is a vague α generalized closed set in (Q ,σ) and then g(f(V)) is a vague α generalized closed set in (R, γ),since g is a Vα generalized closed map. Then (gof)(V) is a vague α generalized closed set. So gof is a vague α generalized closed map.

Definition 3.16: Let f:(P ,τ) →(Q ,σ) be a vague α generalized continuous mapping if $f^{-1}(F)$ is a vague α generalized closed set in P for every vague closed set F in Q.

Example 3.17 : Let P = {a ,b} and Q={c ,d} and G_1={<a,[0.7,0.8] >,< b,[0.6,0.4]>},G_2={<c ,[0.5,0.7] >,< d,[0.4,0.4]>}.Then τ={0,G_1,1} and σ={0,G_2,1} are vague topological spaces on P and Q respectively. Define a mapping f: (P ,τ) → (Q ,σ) by f(a)= c ,f(b) = d. If A={ <c,[0.3,0.5] >,< d ,[0.6,0.6]>} is a vague closed set in Q. So $f^{-1}(F)$={<a, [0.6,0.6] >,< b,[0.3,0.5]>} is a vague α generalized closed set in P. So f is a vague α generalized continuous mapping.

Definition 3.18: Let f:(P ,τ) →(Q ,σ) be a vague M α generalized continuous mapping if $f^{-1}(F)$ is a vague α generalized closed set in P for every vague α generalized closed set F in Q.

Example 3.19: Let P = {a ,b} and Q ={c ,d} and G_1={<a,[0.5,0.7] >,< b,[0.4,0.5]>},G_2={<c,[0.3,0.4] >,< d ,[0.7,0.8]>}.Then τ={0,G_1,1} and σ={0,G_2,1} are vague topological spaces on P and Q respectively. Define a mapping f:(P,τ) → (Q ,σ) by f(a)=c ,f(b)=d. Let F={ <c,[0.5,0.7] >,< d ,[0.4,0.6]>} is a vague α generalized closed set in Q. So $f^{-1}(F)$={<a,[0.5,0.7] >,< b,[0.4,0.6]>} is a vague α generalized closed set in P. So f is a V M α generalised continuous mapping.

Theorem 3.20: A map f : X → Y is a Vαg closed map iff Vαgcl (f(A)) ⊂ fVcl(A).

proof : Let A ⊂ X and f :X →Y is a Vαg closed map then f (Vcl(A)) is a Vαg closed set in Y.
So Vαgcl[f(Vcl(A))]=f(Vcl(A)),since f(A) ⊂ f (Vcl(A)),so
So Vαgcl[f(A)] ⊂ Vαgcl[f(Vcl(A))] = f(Vcl(A)) for all vague subsets A of X.
this implies Vαgcl (f(A)) ⊂ fVcl(A).
Conversely, Let A be any vague closed set in (X ,τ).Then A= Vcl(A) and so f (A) =f (Vcl(A)) ⊇Vαgcl f(A) , by the hypothesis. since f(A) ⊂ Vαgcl[f(A)].
That is f (A) = Vαgcl[f(A)],hence f is a Vαgclosed map.

IV.VAGUE αg HOMEOMORPHISM AND VAGUE M αg HOMEOMORPHISM

Definition 4.1: Let g :(P ,τ) →(Q ,σ) is said to be

1. a vague continuous map if $g^{-1}(F)$ is a vague closed set in (P ,τ) for any vague closed set F of (Q ,σ).
2. a vague α continuous map if $g^{-1}(F)$ is a vague α closed set in (P ,τ) for any vague closed set F of (Q ,σ).
3. a vague α g continuous map if $g^{-1}(F)$ is a vague α generalized closed set in (P ,τ) for any vague closed set F of (Q ,σ). 4. a vague M αg continuous map if $g^{-1}(F)$ is a vague α generalized closed set in (P ,τ) for any vague α generalized closed set F of (Q ,σ).

Definition 4.2: If g :(P ,τ) →(Q ,σ) is both one-one ,onto mapping where(P ,τ) and (Q ,σ) are vague topological spaces, then g is said to be

1.vague homeomorphism if g and g^{-1} are vague continuous mapping.

2.vague α homeomorphism if g and g^{-1} are vague alpha continuous mapping.

3.vague αg homeomorphism if g and g^{-1} are vague alpha generalized continuous mapping.

Definition 4.3: A vague topological space (P ,τ) is said to be a vague ${}_{\alpha g}T_{1/2}$ space if every vague αg closed set is a vague closed set.

Definition 4.4: A vague topological space (P ,τ) is said to be a vague ${}_{\alpha g}T_{1/2}^{\alpha}$ space if every vague αg closed set is a vague α closed set.

Definition 4.5: A vague topological space (P ,τ) is said to be a V ${}_{\alpha g}T_{1/2}^{g}$ space if every vague αg closed set is a vague generalized closed set.

Theorem 4.6: In every vague topological space (P ,τ) the following are obvious.
1. Every vague continuous mapping is a vague α continuous mapping.
2 .Every vague α continuous mapping is a vague αg continuous mapping.

Theorem 4.7: Every vague homeomorphism is a vague αg homeomorphism .
Proof: If g :(P ,τ) → (Q ,σ) be a vague homeomorphism .since g is a vague homeomorphism, g and g^{-1} are vague

continuous mappings .g and g^{-1} are vague αg continuous mapping, since every vague closed set is a vague αg closed set. so f is vague αg homeomorphism.

Remark 4.8: Converse need not be true, seen from the following example,

Example 4.9: Let P={a ,b},Q ={c ,d} , Now τ={0,G_1,1} and σ={0,G_2,1} are vague topological spaces on P and Q respectively ,where G_1={<x,[0.6,0.7],[0.5,0.8]>},G_2={<x,[0.4,0.8],[0.7,0.4]>}.

Now construct a mapping g :(P ,τ) → (Q ,σ) by g(a)=c ,g(b)=d. Then g and g^{-1} are vague αg continuous mapping but not a vague continuous mapping. So g is a vague αg homeomorphism but not a vague homeomorphism.

Theorem 4.10: Every vague α homeomorphism is a vague α g homeomorphism ,but the converse may not be true .

Proof: Let g :(P ,τ) → (Q ,σ) be a vague α homeomorphism .Then g and $g^{-1}$are vague α continuous mappings. This implies g and g^{-1} are vague α g continuous mappings .So g is a vague α g homeomorphism.

Example 4.11: Let P={a ,b},Q={u ,v}.Now τ={0,G_1,1} and σ={0,G_2,1} are vague topological spaces on P and Q respectively, where G_1={<x,[0.6,0.7],[0.5,0.8]>},G_2={<x,[0.4,0.8],[0.7,0.4]>}. Define a mapping g :(P ,τ) → (Q ,σ) by g(a)=u ,g(b)=v. Then g and g^{-1} are vague αg continuous mapping but not a vague α continuous mapping .So g is a vague α g homeomorphism but not a vague α homeomorphism.

Theorem 4.12: Every vague generalized homeomorphism is a vague α g homeomorphism, but the converse is not true in general.

Proof: Let g :(P ,τ) → (Q ,σ) be a vague generalized homeomorphism .Then g and $g^{-1}$are vague generalized continuous mappings. This implies g and g^{-1} are vague α g continuous mappings .So g is a vague α g homeomorphism.

Example 4.13: Let P ={a ,b},Q ={c ,d} and G_1={<a ,[0.6,0.7] >,< b,[0.5,0.8]>},G_2={<c,[0.4,0.8] >,< d ,[0.7,0.4]>}.Then τ={0,G_1,1} and σ={0,G_2,1} are vague topological spaces on P and Q respectively. Define a mapping g :(P ,τ) → (Q ,σ) by g(a)=c ,g(b)=d. Then g and g^{-1} are vague α g continuous mapping but not a vague generalized continuous mapping .So g is a vague αg homeomorphism but not a vague generalized homeomorphism.

Theorem 4.14: Let g :(P ,τ) → (Q ,σ) be a vague α g homeomorphism then g is a vague generalized homeomorphism if P and Q are $V_{\alpha g}T^{g}_{1/2}$ space.

Proof : Let F be a vague closed set in Q .Then g^{-1}(F) is a vague α generalized closed set in P .By hypothesis , Since P is a $V_{\alpha g}T^{g}_{1/2}$ space .This implies g^{-1}(F) is a vague generalized closed set in X .Hence g is a vague generalized continuous mapping .By the hypothesis, g^{-1} is also vague α generalized continuous mapping ,Let E be a vague closed set in P .Then $(g^{-1})^{-1}$(E)=g(E) is a vague α generalized closed set in Q. Since Q is a $V_{\alpha g}T^{g}_{1/2}$ spaces .This implies g(E) is a vague generalized closed set in X. Hence g^{-1} is a vague generalized continuous mapping. Therefore g is a vague generalized homeomorphism.

Theorem 4.16: Let g :(P ,τ) → (Q ,σ) be a bijective mapping .If g is a vague α generalized continuous mapping then the following are equivalent.

1.g is a vague α generalized closed mapping.

2.g is a vague α generalized open mapping.

3.g is a vague α generalized homeomorphism.

Proof :

1 → 2

Let g :(P ,τ) → (Q ,σ) be a bijective mapping and let g be a vague α generalized closed mapping .This implies g^{-1} :(Q, σ) →(P ,τ) is a vague α generalized closed mapping. i .e every vague open set in P there is a vague α generalized open set in Q. Hence g^{-1} is an vague α generalized open mapping.

2 →3

Let g :(P ,τ) → (Q ,σ) be a bijective mapping and let g be a vague α generalized open mapping .This implies g^{-1} :(Q, σ) →(P ,τ) is a vague α generalized continuous mapping. Hence g and g^{-1} are V α generalized continuous mappings .So g is a vague homeomorphism.

3→1

Let g be a vague α generalized homeomorphism .So g and g^{-1} are vague α generalized continuous mappings. Since every vague closed set in P is a vague α generalized closed set in Q. so g is a vague α generalized closed mapping.

Definition 4.17: A map g:(P ,τ) → (Q ,σ) is called a vague α g irresolute if the inverse image of every α g closed set of (Q ,σ) is an αg closed set in (P ,τ).

Definition 4.17: A bijective mapping g :(P ,τ) → (Q ,σ) is called a vague M α generalised homeomorphism if g and g^{-1} are vague α generalised irresolute mappings .

Theorem 4.18: Every vague M α generalized homeomorphism is an vague α generalized homeomorphism .Converse may not be true.

Proof: Suppose g :(P ,τ) → (Q ,σ) is a vague M α generalized homeomorphism .Let F be a vague closed set in Q. This implies F is a vague α generalized closed set in Q. By hypothesis g^{-1}(F) is a vague α generalized closed set in P .Hence g is a vague α generalized continuous mapping .Let E be a vague closed set in P. This implies E is a vague α generalized in P. Then (g^{-1} $)^{-1}$(E) is a vague α generalized

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closed set in Q. Hence g^{-1} is a vague α generalized continuous mapping. So g is a vague α generalized homeomorphism.

Example4.19: Let P ={a ,b},Q ={c ,d} and τ ={0,G_1,1},σ ={0,G_2,1} are topologies on P and Q whereG_1={<a,[0.6,0.7] >,<b,[0.5,0.8]>},G_2={<a,[0.4,0.7] >,<b ,[0.3,0.6]>}. Define a mapping g :(P ,τ) → (Q ,σ) by g(a)=a ,g(b)=b.

g and g^{-1} are the vague α generalized homeomorphism. But g and g^{-1} are not vague M α generalized homeomorphism .since consider the vague α generalized closed set

A ={<x,[0.4,0.7],[0.3,0.5]>}. But g^{-1}(A) =< [0.4,0.7],[0.3,0.5] > is not a vague α generalized closed set.

So f is not a vague M α generalized homeomorphism.

Theorem 4.20:- Suppose g :(P ,τ) → (Q ,σ) is a vague M α generalized homeomorphism then

vαgcl(g^{-1}(F))⊆g^{-1}(vαcl(F)) for every vague closed set F in Q.

Proof : Let F be a vague closed set in Q. Then vαcl(F) is a vague α closed set in Q. This implies every vague α closed set is a vague α generalized closed set in Q. Since the mapping g is vague α g irresolute .So g^{-1}(vαcl(F)) is also a vague α generalized closed set in P. vαgcl(g^{-1}(vαcl (F)))= g^{-1}(vαcl(F)). Now vαgcl(g^{-1}(F)) ⊆ vαgcl(g^{-1}(vαcl (F)))= g^{-1}(vαcl(F)).Hence vαgcl(g^{-1}(F))⊆g^{-1}(vαcl(F)) for every vague closed set F in Q.

Theorem 4.21: Suppose g :(P ,τ) → (Q ,σ) is a vague M α generalized homeomorphism then

vαcl(g^{-1}(F)) =g^{-1}(vαcl(F)) for vague set F in Q.

Proof: Suppose g is a vague M α generalized homeomorphism. so g is a vague α generalized irresolute mapping. Consider a vague set F in Q. vcl(F) is a vague closed set in Q. This implies vcl(F) is a vague α generalized closed set in Q. By hypothesis g^{-1}(vcl(F)) is also a vague α generalized closed set in Q. since g^{-1}(F)⊆ g^{-1}(αcl(F)). vαcl(g^{-1}(F)) ⊆vαcl (g^{-1}(vαcl(F)))=g^{-1}(vαcl(F)).This implies vαcl(g^{-1}(F)) =g^{-1}(vαcl(F)). Since g is a vague M α generalized homeomorphism, g^{-1} :(Q ,σ) →(P ,τ) is a vague α generalized irresolute mapping. Consider a vague set g^{-1}(F) in P. Consider vαcl(g^{-1}(F)) is a vague α generalized closed set in P. Hence vαcl(g^{-1}(F)) is a vague α generalized closed set in P. This implies

$(g^{-1})^{-1}$(g^{-1}(F)) ⊆ $(g^{-1})^{-1}$(vαcl(g^{-1}(F)))=g(vαcl(g^{-1}(F))) .Since g^{-1} is an α generalized irresolute mapping. Hence g^{-1}(vαcl(F)) ⊆g^{-1}(g(vαcl(f^{-1}(F)))) =vαcl (g^{-1}(F)). i .e g^{-1}(vαcl(F)) ⊆ vαcl (g^{-1}(F)). This implies

αcl(g^{-1}(F)) =g^{-1}(αcl(F)).

Theorem 4.22: Composition of two vague M α generalized homeomorphism is a vague M α generalized homeomorphism.

Proof: Let f :(X ,τ) → (y ,σ) and g : (y ,σ) →(Z ,μ) be any two vague M α generalized homeomorphism. Let A be a vague α generalized closed set in z .This implies g^{-1}(A) is a vague α generalized closed set in Y. This implies f^{-1}(g^{-1}(A)) is also vague α generalized closed set in X. Since by the hypothesis, f^{-1}(g^{-1}(A)) is also vague α generalized closed set in X. Hence $(g\ of)^{-1}$(A) is a vague α generalized closed set in X. Hence g of is a vague α generalized irresolute mapping. Hence g of is a vague α generalized irresolute mapping. Now, let f be a vague α generalized closed set in X. f(F) is a vague α generalized closed set in Y. Then by the hypothesis g(f(F)) is a vague α generalized closed set in Z. This implies g of is a vague α generalized irresolute mapping. Hence g of is vague M α generalized homeomorphism. Hence Composition of two vague M α generalized homeomorphism is a vague M α generalized homeomorphism.

Remark 4.23: The composition of two vague α generalized homeomorphisms need not be a vague α generalized homeomorphism in general.

V. VAGUE CONTRA ALPHA GENERALISED CONTINUOUS MAP

Definition 5.1: A map g : (P ,τ) → (Q ,σ) is said to be a vague contra α generalized continuous mapping if g^{-1}(A) is a vague α generalized closed set in (P ,τ) for every vague open set A in (Q ,σ).

Example 5.2: Let P ={a ,b},Q ={c ,d},τ ={0,G_1,1},σ ={ 0,G_2,1}where G_1={<a,[0.6,0.7] >,<b,[0.5,0.8]>},G_2={<c,[0.2,0.9] >, <d,[0.6,0.3]>}.A function g is defined as g(a)=a^c= c and g(b)=b^c = d. So g is a vague contra α generalized continuous mapping.

Theorem 5.3: Every vague contra continuous mapping is a vague contra α generalised continuous mapping but not conversely.

Proof: Let g:(P ,τ) → (Q ,σ) is said to be a vague contra continuous mapping .Let V be an open set in (Q ,σ) this implies g^{-1}(V) is a vague closed set in (P ,τ).Since every vague closed set is a vague α generalized closed set in (P ,τ).so g is vague contra α generalized continuous mapping .

Example 5.4 :-Let X=$\{a,b\}$ and Y=$\{u,v\}$,τ ={0,G_1,1},σ ={ 0,G_2,1}where G_1={<x,[0.6,0.7],[0.5,0.8]>},G_2={<x,[0.2,0.6],[0.6,0.3]>}.f is a vague contra α generalized continuous mapping. But not a vague contra continuous mapping.

Theorem 5.7: Let h :(P ,τ) → (Q ,σ) be a bijective mapping .suppose that one of the following properties hold.

1.h^{-1}(vcl(V))⊆vint(vαgcl(h^{-1}(V))) for each vague set V in Q.

2.vcl(vαgint(h^{-1}(V))) ⊆h^{-1}(vint(V)) for each vague set V in Q.

3.h(vcl(vαgint(K))) ⊆vint(h(K)))) for each vague set K in P.

4. .h(vcl (K)) $\subseteq$vint(h(K)))) for each vague set K in P then h is a vague contra α generalized continuous mapping.

Proof : 1$\Rightarrow$2
This is obvious ,by taking the complements on both sides.
2$\Rightarrow$3
vcl (vαgint(h^{-1}(V)))$\subseteq$ h^{-1}(vint(V)) for each vague set V in Q. Let h(K) =V, implies K=h^{-1}(V)
vcl (vαgint(K)) $\subseteq$ h^{-1}(vint(f(K))).

3 $\Rightarrow$ 4
h(vcl(vαgint(K)) $\subseteq$vint (h(K)).
vαgint(K)=K ,since K is a vαgos in P
So h(vcl(K)) $\subseteq$ vint (h(K)).
Suppose 4 holds, Let K be a vαg open set in P
h^{-1}(K) is a vague open set in P and vαgint(h^{-1}(K)) is a Vαgos in P.
Hence h(vcl(vαgint(h^{-1}(K))) $\subseteq$vint (h(vαgint(h^{-1}(K)))= vint (K)=K
Therefore
vcl(vαgint(h^{-1}(K))=h^{-1}[h(vcl(vint(h^{-1}(K))) $\subseteq$ h^{-1}(K)
Now vcl(vint(h^{-1}(K)) $\subseteq$ vcl(Vαgint(h^{-1}(K)) $\subseteq$$h^{-1}$(K)
Thus h is a VCαG continuous mapping.

Theorem 5.8:-(i) Let f :(P ,τ) $\rightarrow$ (Q ,σ) be a vague contra α generalized continuous map, g :(Q ,σ) $\rightarrow$ (R ,μ) is a vague continuous map. Then g of: (P ,τ) $\rightarrow$ (R ,μ) is a vague contra α generalized continuous map.
(ii) If f is a vague contra α generalized continuous map, g is a vague continuous map then gof is a vαg continuous map.
(iii) If f is a vαg irresolute map, g is a vαg continuous map then gof is a vague contra α generalized continuous map.

Theorem 5.9: A mapping h :(P ,τ) $\rightarrow$ (Q ,σ) be a vague contra α generalized continuous map, where P is a V $\alpha gT1_{/2}$ space iff h^{-1}(vαgcl((V))) $\subseteq$vαgint(h^{-1}(vcl(V)) for every vague set V in Q.

Proof :
Necessity: - Let V be a vague set in Q .Then vcl(V) is a vague closed set in Q .By Hypothesis,
h^{-1}(vαcl((V))) is a vαg open set in P. Since P is a V $_{\alpha G}T_{1/2}$ space, h^{-1}(vcl((V))) is a vague α g open set in P.
h^{-1}(vαgcl((V))) $\subseteq$ h^{-1}(vcl((V))) =vαgint(h^{-1}(vcl(V))).
Sufficiency: - Let V be a vague closed in Q then Vcl(V)=V, By hypothesis,
h^{-1}(vαgcl((V))) $\subseteq$ Vαgint(h^{-1}(vcl(V)) =vαgint(h^{-1}(V)) $\subseteq$ h^{-1}(V).so h is a vague αg open set in P. So h is a vague contra α generalized continuous mapping.

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