Semi-2-Absorbing Ideals Of So-Rings

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Abstract- A partial semiring is a structure having an infinitary partial addition and a binary multiplication, subject to a set of axioms. In this paper we introduce the notions of 2-m-system, semi- 2-absorbing ideal and 2-p-system in so-rings and obtain their characteristics.

Keywords - Ideal, 2-absorbing ideal, semi-2-absorbing ideal, 2-m-system, 2-p-system, commutative so-ring.

I. INTRODUCTION

Partially defined infinitary operations occur in the contexts ranging from integration theory to programming language semantics. The general cardinal algebras studied by Tarski in 1949, Housdorff Topological commutative groups studied by Bourbaki in 1966, $\sum$ - structures studied by Higgs in 1980, sum-ordered partial monoids and sum-ordered partial semirings(so-rings) studied by Arbib, Manes and Benson [2], [4], and streenstrup [13] are some of the algebraic structures of the above type.

G.V.S. Acharyulu [1] and P.V.Srinivasa Rao [10] developed the ideal theory for the sum-ordered partial semirings (so-rings). Continuing this study, in [12] & [7] we introduced notation of 2-absorbing ideals of so-rings and obtained their characteristics in a commutative so-ring. In this paper, we introduce the notion of 2-m-system in so-rings and prove that a proper ideal Q is 2-absorbing ideal if and only if $R \setminus Q$ is a 2-m-system of R. Also we introduce the notions of semi-2-absorbing ideal and 2-p-system in so-rings and prove that a proper ideal Q is semi-2-absorbing ideal if $Q = ABS(Q)$, the set of intersection of all of 2-absorbing ideals of R containing Q.

II.PRELIMINARIES

In this section we collect some important definitions and results for our use in this paper.

2.1. Definition. [4] Let M be a non-empty set M, $\sum$ is a partial addition defined on some, but not necessarily all families $(x_i : i \in I)$ in M. Then the pair (M, $\sum$) said to be a *partial monoid* if they satisify the following axioms:

(i) Unary Sum Axiom. Consider $(x_i : i \in I)$ is a one element family in M, $I = \{j\}$. Then $\sum(x_i : i \in I)$ is defined and equals x_j.

(ii) Partition-Associativity Axiom. Consider $(x_i : i \in I)$ is a family in M, $(I_j : j \in J)$ is a partition of I. Then $(x_i : i \in I)$ is summable if and only if $(x_i : i \in I_j)$ is summable for every $j \in J$ and $\left(\sum(x_i : i \in I_j) : j \in J\right)$ is summable. We write $\sum(x_i : i \in I) = \sum\left(\sum(x_i : i \in I_j) : j \in J\right)$.

2.2. Definition. [4] Let $(R, \sum, ., 1)$ is a quadruple where $(R, \sum)$ is a partial monoid, $(R, ., 1)$ is a monoid with multiplicative operation '.' and unit 1. Then $(R, \sum, ., 1)$ is said to be a *partial semiring* if the additive and multiplicative structures obey the following distributive laws: Suppose $\sum(x_i : i \in I)$ is defined in R. Then for all y in *R*, $\sum(y.x_i : i \in I)$ and $\sum(x_i.y : i \in I)$ are defined and $y.\sum(x_i : i \in I) = \sum(y.x_i : i \in I)$, $\sum(x_i : i \in I).y = \sum(x_i.y : i \in I)$.

2.3. Definition. [4] A partial semiring $(R, \sum, ., 1)$ is said to be *commutative* if $xy = yx$ for every $x, y \in R$.

2.4. Definition. [4] Let $(M, \sum)$ be a partial monoid. The binary relation $\leq$ on a partial monoid is said to be a *sum-ordering* if we have the following axiom: $x \leq y$ if and only if there exists a 'h' in M such that $y = x + h$ for $x, y \in M$.

2.5. Definition. [4] A partial semiring R is said to be a *sum-ordered partial semiring* or *so-ring* if the sum ordering in R is a partial order.

2.6. Example. [4] Let D be a set. Let $Pfn(D,D)$ be the set of all partial functions from D to D be. A family $(x_i : i \in I)$ is summable if and only if for any i, j in I, and $i \neq j$, $dom(x_i) \cap dom(x_j) = \phi$. If $(x_i : i \in I)$ is summable then for any d in D

$$d(\sum_i x_i) = \begin{cases} dx_i, & if \quad d \in dom(x_i) \quad for \quad some \quad (unique) \quad i \in I; \\ & undefined, \quad otherwise. \end{cases}$$

And '.' defined as the usual functional composition. Also the ordering is defined as the extension of functions , unit defined as the identity function on D. Then $(Pfn(D,D), \sum, .)$ is a so-ring.

2.7. Example. [4] Let D be a set. The function $x : D \rightarrow D$ is a multi-function defined as each element in D to an arbitray subset of D. Such multi-functions correspond bijectively to the relation $r \subseteq D \times D$, where $(d,e) \in r$ if and only if $e \in dx$. Let $Mfn(D,D)$ is defined as the set of all multi-functions from D to D, together with $\sum$ defined such that d in D, $d(\sum_i x_i) = \bigcup_i (dx_i)$, and '.' defined as the usual relational composition. That implies for each d in D and for x, y in $Mfn(D,D)$, $d(x.y) = \bigcup(ey : e \in dx)$, and $d1 = \{d\}$. Then $(Mfn(D,D), \sum, .)$ is a so-ring.

2.8. Definition. [4] A So-ring R is said to be *complete* if every family in R is summable.

2.9. Definition. [1] A subset N of a so-ring R is said to be an *ideal* of R if the following conditions are satisified:

(I_1). If $(x_i : i \in N)$ is a summable family in R and $x_i \in N$ $\forall i \in I$ then $\sum(x_i : i \in I) \in N$,

(I_2). If $x \leq y$ and $y \in N$ then $x \in N$,

(I_3). If $x \in N$ and $r \in R$ then $xr, rx \in N$.

Throughout this paper, *R* denotes a commutative so-ring.

III. M-SYSTEM OF SO-RING 'R'

From [12], a 2-absorbing ideal in so-rings is defined as follows:

3.1. Definition. [12] Let J be a proper ideal of a so-ring R. Then J is said to be *2-absorbing* if for any $p, q, r \in R$, $pqr \in J$ implies $pq \in J$ or $qr \in J$ or $qr \in J$.

3.2. Theorem. Let R be a complete so-ring and Q be a proper ideal R. Then the following conditions are equivalent:

(i) Q be a 2-absorbing ideal of R,

(ii) $\{xrysz \mid r,s \in R\} \subseteq Q$ if and only if $xy \in Q$ or $yz \in Q$ or $xz \in Q$.

Proof. (i) $\Rightarrow$ (ii): Assume Q be a 2-absorbing ideal of R. Take $Q' := \{xrysz \mid r,s \in R\}$. If $xy \in Q$ or $yz \in Q$ or $xz \in Q$, then $xrysz \in Q$ $\forall r,s \in R$. Thus $\{xrysz \mid r,s \in R\} \subseteq Q$. Hence $Q' \subseteq Q$. Suppose $Q' \subseteq Q$. Take $X := <x>, Y := <y> \& Z := <z>$. Now we prove that $XYZ \subseteq Q$. Let $a \in XYZ$. Then $a \leq \sum_i x_i y_i z_i$ for $x_i \in <x>, y_i \in <y> \& z_i < z >$. Now $x_i \leq \sum_j r_{i_j} xs_{i_j}, y_i \leq \sum_k r'_{i_k} ys'_{i_k} \& z_i \leq \sum_l r''_{i_l} zs''_{i_l}$ $\forall i \in I, r_{i_j}, s_{i_j}, r'_{i_k}, s'_{i_k}, r''_{i_l}, s''_{i_l} \in R$. $\Rightarrow$ $a \leq \sum_i (\sum_j r_{i_j} xs_{i_j})(\sum_k r'_{i_k} ys'_{i_k})(\sum_l r''_{i_l} zs''_{i_l})$. $\Rightarrow$ $a \leq \sum_i \sum_j \sum_k \sum_l [r_{i_j}(xs_{i_j} r'_{i_k} ys'_{i_k} r''_{i_l} z) s''_{i_l}]$. Since $Q' \subseteq Q$ and Q is an ideal of R, we have $a \in Q$. $\Rightarrow$ $XYZ \subseteq Q$. Since $x \in <x> = X, y \in <y> = Y \& z \in <z> = Z, xyz \in Q$. Since Q is 2-absorbing, $xy \in Q$ or $yz \in Q$ or $xz \in Q$.

(ii) $\Rightarrow$ (i): Suppose $Q' = \{xrysz \mid r,s \in R\} \subseteq Q$ if and only if $xy \in Q$ or $yz \in Q$ or $xz \in Q$. Let $x,y,z \in R \ni xyz \in Q$. Then $(xyz)rs \in Q$ $\forall r,s \in R$. $\Rightarrow$ $\{xrysz \mid r,s \in R\} \subseteq Q$. By assumption, $xy \in Q$ or $yz \in Q$ or $xz \in Q$. Thus Q is a 2-absorbing ideal of R. □

3.3. Definition. Let R be a so-ring and A be a non-empty subset R. Then A is said to be a *2-k-system* if and only if for any $x,y,z \in R \ni xy, yz, xz \in A$, $\exists$ $r,s \in R \ni xrysz \in A$.

3.4. Example. Let $R = \{0,u,v,x,y,1\}$ be a so-ring and the partial addition $\sum$ defined on R by

$$\sum(x_i : i \in I) = \begin{cases} x_j, & if \quad x_i = 0 \quad \forall i \neq j \quad for \quad some \quad j, \\ & undefined, \quad otherwise. \end{cases}$$

and '.' defined by the following table:

.	0	u	v	x	y	1
0	0	0	0	0	0	0
u	0	u	0	0	0	u

v	0	0	v	0	0	v
x	0	0	0	0	0	x
y	0	0	0	0	0	y
1	0	u	v	x	y	1

Consider $A=\{0,u,v\}$. Then A be a non-empty subset of R. Clearly A is a 2-k-system of R.

3.5. Theorem. Let R be a so-ring. Let Q be a proper ideal of R. Then Q is 2-absorbing if and only if $R\setminus Q$ is a 2-k-system.

Proof. Suppose Q is a 2-absorbing ideal of R. Let $x,y,z\in R \ni xy,yz,xz\in R\setminus Q.$ $\Rightarrow$ $xy,yz,xz\notin Q.$ Since Q is 2-absorbing ideal, $xyz\notin Q.$ $\Rightarrow$ $\exists$ 1, $1\in R\ni x.1.y.1.z\in R\setminus Q.$ Hence $R\setminus Q$ is a 2-k-system of R.

Conversely suppose that $R\setminus Q$ is a 2-k-system of R. Let $x,y,z\in R\ni xyz\in Q.$ Suppose $xy\notin Q, yz\notin Q, xz\notin Q.$ $\Rightarrow$ $xy,yz,xz\in R\setminus Q.$ Since $R\setminus Q$ is a 2-m-system, $\exists$ $r,s\in R\ni xrysz\in R\setminus Q.$ $\Rightarrow$ $\exists$ $r,s\in R\ni xrysz\notin Q.$ Since $xyz\in Q,(xyz)rs\in Q.$ $\Rightarrow$ $xrysz\in Q,$ a contradiction. Hence Q is a 2-absorbing ideal of R. □

3.6. Theorem. Let R be a complete so-ring and 'A' be a 2-k-system R. Let Q be an ideal of R which is maximal among all those ideals of R such that $Q\bigcap A=\phi.$ Then Q is a 2-absorbing ideal of R.

Proof. Take $\mathfrak{R}:=\{H\in ideal(R)\,|\,H\bigcap A=\phi\}.$ Clearly Q is a maximal element in $\mathfrak{R}.$ Therefore $\mathfrak{R}$ is non-empty. Let $x,y,z\in R\ni xyz\in Q.$ Suppose that $xy\notin Q, yz\notin Q$ and $xz\notin Q.$ Suppose $xy\notin A$ or $yz\notin A$ or $xz\notin A.$ Then $Q\subset Q\vee\langle xy\rangle$ or $Q\subset Q\vee\langle yz\rangle$ or $Q\subset Q\vee\langle xz\rangle$ and $(Q\vee\langle xy\rangle)\bigcap A=\phi$ or $(Q\vee\langle yz\rangle)\bigcap A=\phi$ or $(Q\vee\langle xz\rangle)\bigcap A=\phi,$ which is a contradiction to the maximality of Q. Hence $xy\in A, yz\in A$ and $xz\in A.$ Since A is 2-k-system of R, $\exists$ $r,s\in R\ni xrysz\in A.$ Since $xyz\in Q, xrysz\in Q.$ $\Rightarrow$ $xrysz\in Q\bigcap A,$ a contradiction to the fact that $Q\bigcap A=\phi.$ Hence $xy\in Q$ or $yz\in Q$ or $xz\in Q.$ Hence Q is a 2-absorbing ideal of R. □

3.7. Theorem. Let R be a complete so-ring and M is a maximal ideal R. Then M is a 2-absobring ideal of R.

Proof. Let M be a maximal ideal of R. Let $x,y,z\in R\ni xyz\in M.$ Take $J':=\{xrysz\,|\,r,s\in R\}\subseteq M.$ Suppose $xy\notin M$, $yz\notin M.$ Then $M\subset M\vee\langle x\rangle\langle y\rangle.\Rightarrow$ $M\subset M\vee\langle y\rangle\langle z\rangle.$ $M\vee\langle xy\rangle=R$ and $M\vee\langle yz\rangle=R.$ $\Rightarrow$ $1\le\sum_i r_i(xy)s_i+m_1\ \&\ 1\le\sum_j r_j(yz)s_j+m_2$ for some $r_i,s_i,r_j,s_j\in R,\ m_1,m_2\in M.$ $\Rightarrow$ $1\le(\sum_i r_i(xy)s_i+m_1)(\sum_j r_j(yz)s_j+m_2)$ $\Rightarrow$ $1\le\sum_i\sum_j r_ir_j(xy)(yz)s_is_j+\sum_i r_i(xy)s_im_2+\sum_j r_j(yz)s_jm_1+m_1m_2.$ $\Rightarrow$ $xz\le(\sum_i\sum_j r_ir_j(xy)(yz)s_is_j)xz+(\sum_i r_i(xy)s_im_2)xz+(\sum_j r_j(yz)s_jm_1)xz+m_1m_2xz\in M.$ $\Rightarrow$ $xz\in M.$ Hence M is a 2-absorbing ideal of R. □

The following example gives the converse need not be true.

3.8. Example. Consider [0, 1] be the unit interval of real numbers . Let $(x_i : i\in I)$ be any family in [0, 1]. Then define $\sum x_i$ as the $\sup x_i$. Also for any x, y in [0, 1], the infimum of {x, y} defined as x.y. Clearly [0, 1] is a so-ring. Now for any $x\in$[0, 1], we have [0, x] is an ideal of [0, 1]. Infact every ideal in [0, 1] is of the form [0, x]. Now for $x=0.5,$ the ideal [0, x] is a 2-absorbing ideal of R, but not a maximal ideal of R.

3.9. Theorem. Let R be a so-ring and I be a 2-absorbing ideal R. Then I contains a minimal 2-absorbing ideal of R.

Proof. Consider $\mathfrak{R}:=\{Q\in ideal(R)\,|\,Q$ is 2-absorbing and $Q\subseteq I\}.$ Clearly $I\in\mathfrak{R}.$ Thus $(\mathfrak{R},\subseteq)$ is a non-empty partially ordered set. Take $\{N_i\,|\,i\in\Delta\}$ be a descending chain of 2-absorbing ideals of R contained in I. Take $N:=\bigcap_{i\in\Delta}N_i.$ Clearly N be an ideal of R with $N\subseteq I.$ Let $x,y,z\in R\ni\{xrysz\,|\,r,s\in R\}\subseteq N.$ Suppose $xy\notin N$ and $yz\notin N.\Rightarrow$ $xy\notin N_k$ and $yz\notin N_l$ for some $k\ge l$ in $\Delta.$ Since $\{N_i\,|\,i\in\Delta\}$ is a descending chain, $xy\notin N_k$ and $yz\notin N_k.$ Since $xy\notin N_k, yz\notin N_k\ \&\ \{xrysz\,|\,r,s\in R\}\subseteq N_k\ \&\ N_k$, we have $xz\in N_k$ (by theorem 3.2). Now $\forall$ $i\le k, N_k\subseteq N_i$ & hence $xz\in N_i$ $\forall$ $i\le k, k\in\Delta.$ Now $\forall$ $i\ge k, N_i\subseteq N_k$ & hence $xy\notin N_i$ and $yz\notin N_i.$ Since $\{xrysz\,|\,r,s\in R\}\subseteq N_i,$ we have $xz\in N_i$ $\forall$

$i \geq k, k \in \Delta. \Rightarrow \quad xz \in N_i \quad \forall \quad i \in \Delta$. Therefore hence $xz \in N = \bigcap_{i \in \Delta} N_i$. Hence N is a 2-absorbing ideal of R. Also $N \in \Re$ and N is a lower bound of $\{N_i \mid i \in \Delta\}$ in $\Re$. So, by zorn's lemma, $\Re$ has a minimal element . Hence the theorem. □

3.10. Theorem. Let R be a complete so-ring and I be an ideal of R. Suppose M is minimal among those ideals of R properly containing I, then $N = \{r \in R \mid rM \subseteq I\}$ is a 2-absorbing ideal of R.

Proof. Clearly $N = \{r \in R \mid rM \subseteq I\} = (I : M)$ is an ideal of R. Take X, Y, Z be ideals of R $\ni XYZ \subseteq N$ then $(XYZ)M \subseteq I$. Suppose $XY \not\subset N$, $YZ \not\subset N$. Then $\exists$ $a, b \in N \ni a \notin XY, b \notin YZ. \Rightarrow$ $(XY)M \not\subset I, (YZ)M \not\subset I.$ Hence $I \subset I \vee (XY)M \subseteq M \,\&\, I \subset I \vee (YZ)M \subseteq M$. By the minimality of M, $I \vee (XY)M = M$ and $I \vee (YZ)M = M. \Rightarrow \quad XZI \vee XZ(XY)M = XZM$ and $XZI \vee XZ(YZ)M = XZM. \Rightarrow$ XZM$\subseteq I \vee I \subseteq I. \Rightarrow$ $(XZ)M \subseteq I. \Rightarrow \quad XZ \subseteq N$. Hence N is a 2-absorbing ideal of R. □

IV. SEMI-2-ABSORBING IDEALS

We define a semi-2-absorbing ideal in so-rings as follows:

4.1. Definition. Let R be a so-ring and I be a proper ideal of R. Then I is said to be a *semi-2-absorbing* ideal of R if for any $x \in R$, $x^3 \in I$ then $x^2 \in I$.

4.2. Remark. Let R be a so-ring and I be a proper ideal of R. If I is a 2-absorbing ideal then I is a semi-2-absorbing ideal of R.

Proof. Let I be a 2-absorbing ideal of R. Let $x \in R \ni x^3 = x.x.x \in I$. Since I is 2-absorbing, $x.x = x^2 \in I$. Thus I is a semi-2-absorbing ideal of R. □

The following example gives the converse need not be true.

4.3. Example. Consider the so-ring $R := Z_{12}$. Let $I =< 0 >$. Then I is a semi-2-absorbing ideal of R. For $2,2,3 \in R$, $2.2.3 = 0 \in I$, $2.2 = 4 \notin I$ and $2.3 = 6 \notin I$. Hence I is not a 2-absorbing ideal of R.

4.4. Theorem. Let R be a complete so-ring and Q be a proper ideal of R. Then the following conditions are equivalent:

(i) Q is a semi-2-absorbing ideal of R,

(ii) $\{xrxsx \mid r, s \in R\} \subseteq Q$ if and only if $x^2 \in Q$.

Proof. (i) $\Rightarrow$ (ii): Suppose Q is a semi-2-absorbing ideal of R. Take $Q' := \{xrxsx \mid r, s \in R\}$. If $x^2 \in Q$. Then $xrxsx \in Q \quad \forall \quad r, s \in R$. Thus $\{xrxsx \mid r, s \in R\} \subseteq Q$. Hence $Q' \subseteq Q$. Suppose $Q' \subseteq Q$. Take $X :=< x >$. Now we prove that $X^3 \subseteq Q$. Let $a \in X^3$. Then $a \leq \sum_i x_i x_i' x_i''$ for $x_i, x_i', x_i'' \in < x >. \Rightarrow$ $x_i \leq \sum_j r_{i_j} xs_{i_j}, x_i' \leq \sum_k r_{i_k}' xs_{i_k}', x_i'' \leq \sum_l r_{i_l}'' xs_{i_l}'', \forall$ $i \in I, r_{i_j}, s_{i_j}, r_{i_k}', s_{i_k}', r_{i_l}'', s_{i_l}'' \in R. \Rightarrow$ $a \leq \sum_i (\sum_j r_{i_j} xs_{i_j})(\sum_k r_{i_k}' xs_{i_k}')(\sum_l r_{i_l}'' xs_{i_l}'')$. Since $Q' \subseteq Q$ and Q is an ideal of R, we have $a \in Q$. Then $X^3 \subseteq Q$. Since $x \in < x >= X$, we have $x^3 \in Q$. Since Q is semi-2-absorbing, $x^2 \in Q$.

(ii) $\Rightarrow$ (i): Suppose $Q' := \{xrxsx \mid r, s \in R\} \subseteq Q$ if and only if $x^2 \in Q$. Let $x \in R \ni x^3 \in Q$. Then $(x.x.x)rs \in Q. \Rightarrow \{xrxsx \mid r, s \in R\} \subseteq Q$. By assumption, $x^2 \in Q$.

Hence Q is a semi-2-absorbing ideal of R. □

4.5. Definition. Let R be a so-ring and **"**A**"** non-empty subset of R. Then A is said to be a *2-l-system* if and only if for any $x \in R \ni x^2 \in A$, $\exists \; r, s \in R \ni xrxsx \in A$.

4.6. Example. Consider the so-ring as in example 3.4., let $A = \{0, u, v\}$ be a non-empty subset of R. Hence A is a 2-l-system of R.

4.7. Remark. Let R be a so-ring. Every 2-k-system of R is a 2-l-system of R.

Proof. Let A be a 2-m-system of R. Let $x \in R$ such that $x^2 \in A$. Clearly $x \in R \ni xx, xx, xx \in A$. Since A is a 2-k-system, $\exists \; r, s \in R \ni xrxsx \in A$. Hence A is a 2-l-system of R. □

4.8. Theorem. Let R be a complete so-ring and Q be a proper ideal of R. Then Q is semi-2-absorbing if and only if $R \setminus Q$ is a 2-l-system of R.

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Proof. Suppose Q is a semi-2-absorbing ideal of R. Let $x \in R \ni x^2 \in R \setminus Q$. Since Q is semi-2-absorbing, $x^3 \notin Q. \Rightarrow \exists$ 1, $1 \in R \ni x.1.x.1.x \in R \setminus Q$. Thus $R \setminus Q$ is a 2-l-system.

Conversly suppose that $R \setminus Q$ is a 2-l-system of R. Consider $x \in R \ni x.x.x \in Q$. Suppose $x^2 \notin Q$. Then $x.x \in R \setminus Q$. Since $R \setminus Q$ is a 2-p-system of R, $\exists$ $r, s \in R \ni xrxsx \in R \setminus Q. \Rightarrow \exists$ $r, s \in R \ni xrxsx \notin Q$. Since $x.x.x \in Q, (x.x.x)r.s \in Q. \Rightarrow$ $xrxsx \in Q$, a contradiction. Therefore $x^2 \in Q$. Hence Q is a semi-2-absorbing ideal of R. □

4.9. Remark. Arbitray union of a family of 2-l-systems of a so-ring R is again a 2-l-system of R.

Proof. Let $\{A_i \mid i \in I\}$ be the family of 2-p-systems of R. Take $A := \bigcup_{i \in I} A_i$. Let $a \in R \ni a^2 \in A. \Rightarrow$ $a^2 \in A_i$ for some $i \in I$. Since A_i is 2-p-system of R, $\exists$ $r, s \in R \ni arasa \in A_i, i \in I. \Rightarrow$ $arasa \in \bigcup_{i \in I} A_i$.

Therefore $A := \bigcup_{i \in I} A_i$ is a 2-p-system of R. □

4.10. Theorem. A non-empty subset H of a complete so-ring R is a 2-p-system if it is the union of 2-m-systems.

Proof. Suppose H is the union of 2-m-systems. i.e., $H = \bigcup_{i \in \Delta} N_i$ where each N_i is a 2-m-system. Since every 2-m-system is a 2-p-system & union of 2-p-systems is again a 2-p-system, H is a 2-p-system. □

We denote ABS(R) is the set of all 2-absorbing ideals of R. We denote ABS(I) is the intersection of all 2-absorbing ideals of R containing I. i.e., $ABS(I) = \bigcap\{J \in ABS(R) \mid I \subset J\}$.

4.11. Theorem. A proper ideal Q of a complete so-ring R is semi-2-absorbing if $Q = ABS(Q)$.

Proof. Suppose $Q = ABS(Q) = \bigcap\{L \mid L$ is 2-absorbing, $Q \subseteq L\}$. $\Rightarrow$ $R \setminus Q = R \setminus \bigcap\{L \in ABS(R) \mid Q \subseteq L\} = \bigcup\{R \setminus L \mid L \in ABS(R),$ $Q \subseteq L\}$. Since L is 2-absorbing, $R \setminus L$ is a 2-m-system. Then by theorem 4.10., $R \setminus Q$ is a 2-p-system of R. Hence by theorem 4.9., Q is a semi-2-absorbing ideal of R. □

4.12. Remark. Let R be a complete so-ring and J, K be any two ideals of R. Then

$J \subseteq K. \Rightarrow ABS(J) \subseteq ABS(K)$.

Proof. (i) Suppose $J \subseteq K$. Then $\{Q \in ABS(R) \mid J \subseteq Q\} \supseteq \{Q \in ABS(R) \mid K \subseteq Q\}. \Rightarrow \bigcap\{Q \in ABS($ Hence $ABS(J) \subseteq ABS(K)$. □

V. CONCLUSION

In this paper, we introduced the notion of 2-m-system in so-rings and proved that a proper ideal Q is 2-absorbing ideal if and only if $R \setminus Q$ is a 2-m-system of R. Also we introduced the notions of semi-2-absorbing ideal and 2-p-system in so-rings and proved that a proper ideal Q is semi-2-absorbing ideal if $Q = ABS(Q)$, the set of intersection of all of 2-absorbing ideals of R containing Q.

REFERENCES

[1] Acharyulu, G.V.S., "A Study of Sum-Ordered Partial Semirings", Doctoral thesis, Andhra University, 1992.

[2] Arbib, M.A., Manes, E.G., "Partially Additive Categories and Flow-diagram Semantics", Journal of Algebra. Vol. 62, pp. 203-227, 1980.

[3] Chaudhari J.N., "2-absorbing Ideals in Semirings", International Journal of Algera. Vol. 6(6), pp. 265-270 , 2012.

[4] Manes, E.G., and Benson, D.B., "The Inverse Semigroup of a Sum-Ordered Partial Semirings", Semigroup Forum. Vol. 31, pp. 129-152 1985.

[5] Prathibha Kumar, Manish Kant Dubey and Poonam Sarohe., "Some results on 2-absorbing ideals in Commutative Semirings", Journal of Mathematics and Applications. Vol. 38, pp. 77-84, 2015.

[6] Ravi Babu, N., Pradeep Kumar, T.V., and Srinivasa Rao, P.V., "weakly 2-absorbing ideals of so-rings", International Journal of Scientific & Innovative Mathematical Research, Vol. 5(7), pp. 29-35, 2017.

[7] Ravi Babu, N., Pradeep Kumar, T.V., and Srinivasa Rao, P.V., "2-absorbing ideals in so-rings" (communicated to International Journal of Pure and Applied Mathematics).

[8] Ravi Babu, N., Pradeep Kumar, T.V., and Srinivasa Rao, P.V., "weakly 2-absorbing primary ideals in so-rings" (communicated to International Journal of Pure and Applied Mathematics).

[9] Ravi Babu, N., Pradeep Kumar, T.V., and Srinivasa Rao, P.V., "2-absorbing primary ideals in so-rings" (communicated to Jordan Journal of Mathematics & Statistics).

[10] Srinivasa Rao, P.V., "Ideal Theory of Sum-ordered Partial Semirings", Doctoral thesis, Acharya Nagarjuna University, 2011.

[11] Srinivasa Rao, P.V., "Ideals of Sum-ordered Semirings", International Journal of Computational Cognition (IJCC). Vol. 7(2), pp. 59-64, 2009.

[12] Srinivasa Reddy, M., Amarendra Babu, V., Srinivasa Rao, P.V., "2-absorbing Subsemimodules of Partial Semimodules", Gen.Math.Notes, Vol. 23(2), pp. 43-50, 2014.

[13] Streenstrup, M.E., "Sum-ordered Partial Semirings", Doctoral thesis, Department of computer and Information Science, Graduate school of the University of Massachusetts, 1985.