Weakly - R_0 Type Spaces On Pre-Semi Closed Sets In Topological Ordered Spaces

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Abstract: In this paper we define some new separation axioms of type Weakly-R_0 spaces and we introduced weakly ps-i-R_0, weakly ps-d-R_0, weakly ps-b-R_0 spaces for topological ordered spaces and we verify the relationship between these spaces to other spaces in topological ordered spaces.

Keywords* — *weakly ps-i-R_0, weakly ps-d-R_0, weakly ps-b-R_0 and topological ordered spaces.

I. INTRODUCTION

A topological ordered space [8] is a triple $(X,\tau,\leq)$, where τ is topology on X and $\leq$ is a partial ordered on X. Let $(X,\tau,\leq)$ be a topological ordered spaces, for any $x \in X$, $[x,\rightarrow] = \{y \in X/\ x\leq y\}$ and $[\leftarrow,x] = \{y \in X/\ y\leq x\}$. A sub set A of a topological ordered space$(X,\tau,\leq)$ is said to be increasing [8] if A= i(A) and decreasing [8] if A= d(A),where $i(A) = U_{a\in A}\,[a,\rightarrow]$ and $d(A) = U_{a\in A}[\leftarrow,a]$. A sub set A of a topological ordered space $(X,\tau,\leq)$ is said to be balanced [8] if it is both increasing and decreasing. Let (X,τ) be a topological space and A be a sub set of X, the interior of A is (denoted by int (A)) is the union of all open sets of A and closure of A (denoted by cl(A)) is the intersection of all closed super sets of A,C(A) denotes the complement of A. Weakly -R_0 space was introduced by G. Di Maio [4] in 1984 now in the present paper we introduced weakly ps-i-R_0 , weakly ps-d-R_0, weakly ps-b-R_0 spaces for topological ordered spaces.

II. PRELIMINARIES

Definition 2.1: A sub set A of a topological space (X,τ) is called

(i) semi-open set [5] if $A \subseteq cl(int(A))$ and semi-closed if $int(cl(A)) \subseteq A$.

(ii) α-open set [10] if $A \subseteq int(cl(int(A)))$ and α-closed set[2] if $cl(int(cl(A))) \subseteq A$.

(iii) pre-open set [7] $A \subseteq int(cl(A))$ and pre-closed if $cl(int(A)) \subseteq A$.

(iv) regular open set if A= int(cl(A)) and regular closed set if A= cl(int(A)).

Definition 2.2: A sub set A of a topological space (X,τ) is called

(i) Pre-semi-closed set (briefly ps-closed) [12] if $spcl(A)\subseteq U$ and U is g-open in (X,τ).

(ii) Ψ- closed [11] if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is sg-open in (X,τ).

(iii) Generalized α-closed set(briefly gα-closed) [6] if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is α-open in (X,τ).

Definition 2.3: [10] A sub set A of a topological ordered space $(X,\tau,\leq)$ is called

(i) An increasing (resp .decreasing, balanced) semi-closed set if it is both increasing (resp. decreasing, balanced) and semi closed set.

(ii) An increasing (resp .decreasing, balanced) α-closed set if it is both increasing (resp .decreasing, balanced) and α-closed set.

(iii) An increasing (resp .decreasing, balanced) pre-closed set if it is both increasing (resp .decreasing, balanced) and pre-closed set.

(iv) An increasing (resp .decreasing, balanced) regular-closed set if it is both increasing (resp .decreasing, balanced) and regular-closed set.

Definition 2.4:[10] A sub set A of a topological ordered space $(X,\tau,\leq)$ is called

(i) psi (resp.psd, psb) closed set if it is both pre-semi and increasing (resp .decreasing, balanced) closed set.

(ii) Ψi (resp.Ψd, Ψb) closed set if it is both increasing (resp .decreasing, balanced) and Ψ-closed set.

(iii) gαi (resp. gαd, gαb) closed set if it is both increasing (resp .decreasing, balanced) and gα-closed.

Definition 2.5: A topological space (X,τ) is called

(i) R_0 space [2] if cl $\{x\} \in G$ whenever $x \in G \in \tau$.

(ii) weakly -R_0 [4] if $\cap_{x\in X}$ cl $\{x\} = \phi$.

(iii) weakly-i-semi-R_0 space [1] (resp.d-semi-R_0, b-semi-R_0) if $\cap_{x\in X}$ iscl$\{x\} = \phi$ (resp. $\cap_{x\in X}$ dscl$\{x\} = \phi$, $\cap_{x\in X}$ bscl$\{x\} = \phi$)

III. W-R0 TYPE SPACES ON PSI (RESP. PSD, PSB) CLOSED SETS IN TOPOLOGICAL ORDERED SPACES

We introduce the following definitions.

Definition 3.1: A topological ordered space $(X,\tau,\leq)$ is called

(i) Weakly ps-i-R_0 (resp. ps-d-R_0, ps-b-R_0) space if $\cap_{x\in X}$ psicl$\{x\} = \phi$ (resp. $\cap_{x\in X}$ psdcl$\{x\} = \phi$, $\cap_{x\in X}$ psbcl$\{x\} = \phi$).

(ii) Weakly Ψ-i-R_0 (resp. Ψ-d-R_0, Ψ-b-R_0) space if $\cap_{x\in X}$ Ψicl$\{x\} = \phi$ (resp. $\cap_{x\in X}$ Ψdcl$\{x\} = \phi$, $\cap_{x\in X}$ Ψbcl$\{x\} = \phi$).

(iii) Waekly gα-i-R_0 (resp. gα-d-R_0, gα-b-R_0) space if $\cap_{x\in X}$ gαicl$\{x\}$ = ϕ (resp. $\cap_{x\in X}$ αdcl$\{x\}$ = ϕ, $\cap_{x\in X}$ gαbcl$\{x\}$ = ϕ).

(iv) Weakly r-i-R_0 (resp. r-d-R_0, r-b-R_0) spaces if $\cap_{x\in X}$ ricl$\{x\}$ = ϕ (resp. $\cap_{x\in X}$ rdcl$\{x\}$ = ϕ, $\cap_{x\in X}$ rbcl$\{x\}$ = ϕ).

(v) Weakly i-α-R_0 (resp. d-α-R_0, b-α-R_0) space if $\cap_{x\in X}$ iαcl$\{x\}$ = ϕ (resp. $\cap_{x\in X}$ dαcl$\{x\}$ = ϕ, $\cap_{x\in X}$ bαcl$\{x\}$ = ϕ).

(vi) Weakly i-R_0 (resp. d-R_0 , b-R_0) space if $\cap_{x\in X}$ icl$\{x\}$ = ϕ (resp. $\cap_{x\in X}$ dcl$\{x\}$ = ϕ, $\cap_{x\in X}$ bcl$\{x\}$ = ϕ).

(vii) weakly i-pre-R_0 (resp. d-pre-R_0, b-pre-R_0) space if $\cap_{x\in X}$ ipcl$\{x\}$ = ϕ (resp. $\cap_{x\in X}$ dpcl$\{x\}$ = ϕ, $\cap_{x\in X}$ bpcl$\{x\}$ = ϕ).

Theorem 3.2:

(i) Every weakly Ψ-i-R_0 space is weakly ps-i-R_0 space.

(ii) Every weakly i-semi-R_0 space is weakly ps-i-R_0 space.

(iii) Every weakly i-α-R_0 space is weakly ps-i-R_0 space.

(iv) Every weakly i-R_0 space is weakly ps-i-R_0 space.

(v) Every weakly i-pre-R_0 space is weakly ps-i-R_0 space.

(vi) Every weakly gα-i-R_0 space is weakly ps-i-R_0 space.

(vii) Every weakly r-i-R_0 space is weakly ps-i-R_0 space.

Proof: (i)

Let (X,τ,≤) be any weakly Ψ-i-R_0 space .

⇒ $\cap_{x\in X}$ Ψicl$\{x\}$ = ϕ. Every Ψi-closed set is psi-closed set.

We have psicl$\{x\}$ ⊆ Ψicl$\{x\}$, ∀ x∈ X.

⇒ $\cap_{x\in X}$ psicl$\{x\}$ ⊆ $\cap_{x\in X}$ Ψicl$\{x\}$. Since $\cap_{x\in X}$ Ψicl$\{x\}$ = ϕ, we have $\cap_{x\in X}$ psicl$\{x\}$ = ϕ. Therefore (X,τ, ≤) is weakly ps-i-R_0 space. Hence every weakly Ψ-i-R_0 space is weakly ps-i-R_0 space.

Similarly we can prove (ii), (iii), (iv), (v), (vi), (vii).

Remark 3.3: The converse of the above theorem is not true in general it can be seen in the following example.

Example 3.4:

(i) Let X = {a, b, c}, τ = {ϕ, X, {a}, {a, c}} and ≤ = {(a, a), (b, b), (c, c), (b, a)} be a topological ordered space. Then (X,τ, ≤) is a weekly ps-i-R_0 space but not weakly Ψ-i-R_0 space.

(ii) Let X = {a, b, c}, τ = {ϕ, X, {a}, {a, c}} and ≤ = {(a, a), (b, b), (c, c), (b, a)} be a topological ordered space. Then (X,τ ,≤) is a weekly ps-i-R_0 space but not weakly i-semi-R_0 space.

(iii) Let X = {a, b, c}, τ = {ϕ, X, {a}, {b}, {a, b}} and ≤ = {(a, a), (b, b), (c, c)} be a topological ordered space. Then (X,τ ,≤) is a weekly ps-i-R_0 space but not weakly i-α-R_0 space.

(iv) Let X = {a, b, c}, τ = {ϕ, X, {a}, {b}, {a, b}} and ≤ = {(a, a), (b, b), (c, c),} be a topological ordered space. Then (X,τ ,≤) is a weekly ps-i-R_0 space but not weakly i-R_0 space.

(v) Let X = {a, b, c}, τ = {ϕ, X, {a}, {b}, {a, b}} and ≤ = {(a, a), (b, b), (c, c),} be a topological ordered space. Then (X,τ ,≤) is a weekly ps-i-R_0 space but not weakly i-pre-R_0 space.

(vi) Let X = {a, b, c}, τ = {ϕ, X, {a}, {b}, {a, b}} and ≤ = {(a, a), (b, b), (c, c), (a, b), (a, c)} be a topological ordered space. Then (X,τ ,≤) is a weekly ps-i-R_0 space but not weakly gα-i-R_0 space.

(vii) Let X = {a, b, c}, τ = {ϕ, X, {a}, {a, c}} and ≤ = {(a, a), (b, b), (c, c),} be a topological ordered space. Then (X,τ ,≤) is a weekly ps-i-R_0 space but not weakly r-i-R_0 space.

Example 3.5:

(i) Every weakly Ψ-d-R_0 space is weakly ps-d-R_0 space.

(ii) Every weakly d-semi-R_0 space is weakly ps-d-R_0 space.

(iii) Every weakly d-α-R_0 space is weakly ps-d-R_0 space.

(iv) Every weakly d-R_0 space is weakly ps-d-R_0 space.

(v) Every weakly d-pre-R_0 space is weakly ps-d-R_0 space.

(vi) Every weakly gα-d-R_0 space is weakly ps-d-R_0 space.

(vii) Every weakly r-d-R_0 space is weakly ps-d-R_0 space.

Proof: (i)

Let (X,τ,≤) be any weakly Ψ-d-R_0 space .

⇒ $\cap_{x\in X}$ Ψdcl$\{x\}$ = ϕ. Every Ψd-closed set is psd-closed set.

We have psdcl$\{x\}$ ⊆ Ψdcl$\{x\}$, ∀ x∈ X.

⇒ $\cap_{x\in X}$ psdcl$\{x\}$ ⊆ $\cap_{x\in X}$ Ψdcl$\{x\}$. Since $\cap_{x\in X}$ Ψdcl$\{x\}$ = ϕ, we have $\cap_{x\in X}$ psdcl$\{x\}$ = ϕ. Therefore (X,τ, ≤) is weakly ps-d-R_0 space. Hence every weakly Ψ-d-R_0 space is weakly ps-d-R_0 space.

Similarly we can prove (ii), (iii), (iv), (v), (vi), (vii).

Remark 3.6: The converse of the above theorem is not true in general it can be seen in the following example.

Example 3.7:

(i) Let X = {a, b, c}, τ = {ϕ, X, {a}, {a, c}} and ≤ = {(a, a), (b, b), (c, c), (a, c), (b, c)} be a topological ordered space. Then (X,τ, ≤) is a waekly ps-d-R_0 space but not weakly Ψ-d-R_0 space.

(ii) Let X = {a, b, c}, τ = {ϕ, X, {a}, {a, c}} and ≤ = {(a, a), (b, b), (c, c), (a, c), (b, c)} be a topological ordered space. Then (X,τ,≤) is a weekly ps-d-R_0 space but not weakly d-semi-R_0 space.

(iii) Let X = {a, b, c}, τ = {ϕ, X, {a}, {b}, {a, b}} and ≤ = {(a, a), (b, b), (c, c), (a, b), (c, b)} be a topological ordered space. Then (X,τ, ≤) is a weakly ps-d-R_0 space but not weakly d-α-R_0 space.

(iv) Let X = {a, b, c}, τ = {ϕ, X, {a}, {a, c}} and ≤ = {(a, a), (b, b), (c, c), (a, b), (c, b)} be a topological ordered space. Then (X,τ, ≤) is a weakly ps-d-R_0 space but not weakly d-R_0 space.

(v) Let X = {a, b, c}, τ = {ϕ, X, {a}, {b}, {a, b}} and ≤ = {(a, a), (b, b), (c, c), (a, c), (b, c)} be a topological ordered space. Then (X,τ ,≤) is a weakly ps-d-R_0 space but not weakly d-pre-R_0 space.

(vi) Let X = {a, b, c}, τ = {ϕ, X, {a}, {b}, {a, b}} and ≤ = {(a, a), (b, b), (c, c), (a, b), (c, b)} be a topological ordered space. Then (X,τ ,≤) is a weakly ps-d-R_0 space but not weakly gα-d-R_0 space.

Let X = {a, b, c}, τ = {ϕ, X, {a}, {b}, {a, b}} and ≤ = {(a, a), (b, b), (c, c), (a, b), (c, b)} be a topological ordered space. Then (X,τ ,≤) is a weakly ps-d-R_0 space but not weakly r-d-R_0 space.

Theorem 3.8:

(i) Every weakly Ψ-b-R_0 space is weakly ps-b-R_0 space.

(ii) Every weakly b-semi-R_0 space is weakly ps-b-R_0 space.

(iii) Every weakly b-α-R_0 space is weakly ps-b-R_0 space.

(iv) Every weakly b-R_0 space is weakly ps-b-R_0 space.

(v) Every weakly b-pre-R_0 space is weakly ps-b-R_0 space.

(vi) Every weakly gα-b-R_0 space is weakly ps-b-R_0 space.

(vii) Every weakly r-b-R_0 space is weakly ps-b-R_0 space.

Proof: (i)

Let (X,τ,≤) be any weakly Ψ-b-R_0 space .

$\Rightarrow \cap_{x\in X} \Psi bcl\{x\} = \phi$. Every Ψb-closed set is psb-closed set.

We have $psbcl\{x\} \subseteq \Psi bcl\{x\}$, $\forall\, x \in X$.

$\Rightarrow \cap_{x\in X} psbcl\{x\} \subseteq \cap_{x\in X} \Psi bcl\{x\}$. Since $\cap_{x\in X} \Psi bcl\{x\} = \phi$, we have $\cap_{x\in X} psbcl\{x\} = \phi$. Therefore (X,τ, ≤) is weakly ps-b-R_0 space. Hence every weakly Ψ-b-R_0 space is weakly ps-b-R_0 space.

Similarly we can prove (ii), (iii), (iv), (v), (vi), (vii).

Remark 3.9: The converse of the above theorem is not true in general it can be seen in the following example.

Example 3.10:

(i) Let X = {a, b, c}, τ = {ϕ, X, {a}, {a, c}} and ≤ = {(a, a), (b, b), (c, c), (a, c)} be a topological ordered space. Then (X,τ, ≤) is a waekly ps-b-R_0 space but not weakly Ψ-b-R_0 space.

(ii) Let X = {a, b, c}, τ = {ϕ, X, {a}, {a, c}} and ≤ = {(a, a), (b, b), (c, c), (b, a)} be a topological ordered space. Then (X,τ,≤) is a weakly ps-b-R_0 space but not weakly b-semi-R_0 space.

(iii) Let X = {a, b, c}, τ = {ϕ, X, {a}, {a, c}} and ≤ = {(a, a), (b, b), (c, c), (a, c)} be a topological ordered space. Then (X,τ, ≤) is a weakly ps-b-R_0 space but not weakly b-α-R_0 space.

(iv) Let X = {a, b, c}, τ = {ϕ, X, {a}, {b}, {a, b}} and ≤ = {(a, a), (b, b), (c, c), (a, c)} be a topological ordered space. Then (X,τ, ≤) is a weakly ps-b-R_0 space but not weakly b-R_0 space.

(v) Let X = {a, b, c}, τ = {ϕ, X, {a}, {a, c}} and ≤ = {(a, a), (b, b), (c, c), (b, a)} be a topological ordered space. Then (X,τ ,≤) is a weakly ps-b-R_0 space but not weakly b-pre-R_0 space.

(vi) Let X = {a, b, c}, τ = {ϕ, X, {a}, {b}, {a, b}} and ≤ = {(a, a), (b, b), (c, c), (a, c)} be a topological ordered space. Then (X,τ ,≤) is a weakly ps-b-R_0 space but not weakly gα-b-R_0 space.

(vii) Let X = {a, b, c}, τ = {ϕ, X, {a}, {b}, {a, b}} and ≤ = {(a, a), (b, b), (c, c), (a, c)} be a topological ordered space. Then (X,τ ,≤) is a weakly ps-b-R_0 space but not weakly r-b-R_0 space.

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