

Weakly- R_0 Type Spaces on Pre-Semi Closed Sets in Topological Ordered Spaces

¹Dr. V. Amarendra Babu, ²P. Rajasekhar

¹Assistant Professor, ²Research scholar, ^{1,2}Department of mathematics, Acharya Nagarjuna University, Nagarjuna Nagar, Guntur (District) A.P, India.

¹amarendravelisela@gmail.com, ²pathururaja1@gmail.com

Abstract: In this paper we define some new separation axioms of type Weakly- R_0 spaces and we introduced weakly ps-i- R_0 , weakly ps-d- R_0 , weakly ps-b- R_0 spaces for topological ordered spaces and we verify the relationship between these spaces to other spaces in topological ordered spaces.

Keywords — weakly ps-i- R_0 , weakly ps-d- R_0 , weakly ps-b- R_0 and topological ordered spaces.

I. INTRODUCTION

A topological ordered space [8] is a triple (X, τ, \leq) , where τ is topology on X and \leq is a partial ordered on X . Let (X, τ, \leq) be a topological ordered spaces, for any $x \in X$, $[x, \rightarrow] = \{y \in X / x \leq y\}$ and $[\leftarrow, x] = \{y \in X / y \leq x\}$. A sub set A of a topological ordered space (X, τ, \leq) is said to be increasing [8] if $A = i(A)$ and decreasing [8] if $A = d(A)$, where $i(A) = \bigcup_{a \in A} [a, \rightarrow]$ and $d(A) = \bigcup_{a \in A} [\leftarrow, a]$. A sub set A of a topological ordered space (X, τ, \leq) is said to be balanced [8] if it is both increasing and decreasing. Let (X, τ) be a topological space and A be a sub set of X , the interior of A is (denoted by $\text{int}(A)$) is the union of all open sets of A and closure of A (denoted by $\text{cl}(A)$) is the intersection of all closed super sets of A , $C(A)$ denotes the complement of A . Weakly - R_0 space was introduced by G. Di Maio [4] in 1984 now in the present paper we introduced weakly ps-i- R_0 , weakly ps-d- R_0 , weakly ps-b- R_0 spaces for topological ordered spaces.

II. PRELIMINARIES

Definition 2.1: A sub set A of a topological space (X, τ) is called

- (i) semi-open set [5] if $A \subseteq \text{cl}(\text{int}(A))$ and semi-closed if $\text{int}(\text{cl}(A)) \subseteq A$.
- (ii) α -open set [10] if $A \subseteq \text{int}(\text{cl}(\text{int}(A)))$ and α -closed set [2] if $\text{cl}(\text{int}(\text{cl}(A))) \subseteq A$.
- (iii) pre-open set [7] $A \subseteq \text{int}(\text{cl}(A))$ and pre-closed if $\text{cl}(\text{int}(A)) \subseteq A$.
- (iv) regular open set if $A = \text{int}(\text{cl}(A))$ and regular closed set if $A = \text{cl}(\text{int}(A))$.

Definition 2.2: A sub set A of a topological space (X, τ) is called

- (i) Pre-semi-closed set (briefly ps-closed) [12] if $\text{spcl}(A) \subseteq U$ and U is g -open in (X, τ) .
- (ii) Ψ - closed [11] if $\text{scl}(A) \subseteq U$ whenever $A \subseteq U$ and U is sg -open in (X, τ) .
- (iii) Generalized α -closed set (briefly $g\alpha$ -closed) [6] if $\alpha \text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is α -open in (X, τ) .

Definition 2.3: [10] A sub set A of a topological ordered space (X, τ, \leq) is called

- (i) An increasing (resp .decreasing, balanced) semi-closed set if it is both increasing (resp.decreasing, balanced) and semi closed set.
- (ii) An increasing (resp .decreasing, balanced) α -closed set if it is both increasing (resp .decreasing, balanced) and α -closed set.
- (iii) An increasing (resp .decreasing, balanced) pre-closed set if it is both increasing (resp .decreasing, balanced) and pre-closed set.
- (iv) An increasing (resp .decreasing, balanced) regular-closed set if it is both increasing (resp .decreasing, balanced) and regular-closed set.

Definition 2.4:[10] A sub set A of a topological ordered space (X, τ, \leq) is called

- (i) psi (resp.psd, psb) closed set if it is both pre-semi and increasing (resp .decreasing, balanced) closed set.
- (ii) Ψi (resp. Ψd , Ψb) closed set if it is both increasing (resp .decreasing, balanced) and Ψ -closed set.
- (iii) gai (resp. gad , gab) closed set if it is both increasing (resp .decreasing, balanced) and $g\alpha$ -closed.

Definition 2.5: A topological space (X, τ) is called

- (i) R_0 space [2] if $\text{cl} \{x\} \in G$ whenever $x \in G \in \tau$.
- (ii) weakly - R_0 [4] if $\bigcap_{x \in X} \text{cl} \{x\} = \phi$.

(iii) weakly-i-semi- R_0 space [1] (resp.d-semi- R_0 , b-semi- R_0) if $\cap_{x \in X} \text{iscl}\{x\} = \phi$ (resp. $\cap_{x \in X} \text{dscl}\{x\} = \phi$, $\cap_{x \in X} \text{bscl}\{x\} = \phi$)

III. W- R_0 TYPE SPACES ON PSI (RESP. PSD, PSB) CLOSED SETS IN TOPOLOGICAL ORDERED SPACES

We introduce the following definitions.

Definition 3.1: A topological ordered space (X, τ, \leq) is called

(i) Weakly ps-i- R_0 (resp. ps-d- R_0 , ps-b- R_0) space if $\cap_{x \in X} \text{psicl}\{x\} = \phi$ (resp. $\cap_{x \in X} \text{psdcl}\{x\} = \phi$, $\cap_{x \in X} \text{psbcl}\{x\} = \phi$).

(ii) Weakly Ψ -i- R_0 (resp. Ψ -d- R_0 , Ψ -b- R_0) space if $\cap_{x \in X} \Psi \text{icl}\{x\} = \phi$ (resp. $\cap_{x \in X} \Psi \text{dcl}\{x\} = \phi$, $\cap_{x \in X} \Psi \text{bcl}\{x\} = \phi$).

(iii) Weakly $g\alpha$ -i- R_0 (resp. $g\alpha$ -d- R_0 , $g\alpha$ -b- R_0) space if $\cap_{x \in X} g\alpha \text{icl}\{x\} = \phi$ (resp. $\cap_{x \in X} g\alpha \text{dcl}\{x\} = \phi$, $\cap_{x \in X} g\alpha \text{bcl}\{x\} = \phi$).

(iv) Weakly r-i- R_0 (resp. r-d- R_0 , r-b- R_0) spaces if $\cap_{x \in X} \text{ricl}\{x\} = \phi$ (resp. $\cap_{x \in X} \text{rdcl}\{x\} = \phi$, $\cap_{x \in X} \text{rbcl}\{x\} = \phi$).

(v) Weakly i- α - R_0 (resp. d- α - R_0 , b- α - R_0) space if $\cap_{x \in X} i\alpha \text{icl}\{x\} = \phi$ (resp. $\cap_{x \in X} d\alpha \text{icl}\{x\} = \phi$, $\cap_{x \in X} b\alpha \text{icl}\{x\} = \phi$).

(vi) Weakly i- R_0 (resp. d- R_0 , b- R_0) space if $\cap_{x \in X} \text{icl}\{x\} = \phi$ (resp. $\cap_{x \in X} \text{dcl}\{x\} = \phi$, $\cap_{x \in X} \text{bcl}\{x\} = \phi$).

(vii) weakly i-pre- R_0 (resp. d-pre- R_0 , b-pre- R_0) space if $\cap_{x \in X} \text{ipcl}\{x\} = \phi$ (resp. $\cap_{x \in X} \text{dpcl}\{x\} = \phi$, $\cap_{x \in X} \text{bpcl}\{x\} = \phi$).

Theorem 3.2:

- (i) Every weakly Ψ -i- R_0 space is weakly ps-i- R_0 space.
- (ii) Every weakly i-semi- R_0 space is weakly ps-i- R_0 space.
- (iii) Every weakly i- α - R_0 space is weakly ps-i- R_0 space.
- (iv) Every weakly i- R_0 space is weakly ps-i- R_0 space.
- (v) Every weakly i-pre- R_0 space is weakly ps-i- R_0 space.
- (vi) Every weakly $g\alpha$ -i- R_0 space is weakly ps-i- R_0 space.
- (vii) Every weakly r-i- R_0 space is weakly ps-i- R_0 space.

Proof: (i)

Let (X, τ, \leq) be any weakly Ψ -i- R_0 space .

$\Rightarrow \cap_{x \in X} \Psi \text{icl}\{x\} = \phi$. Every Ψ i-closed set is psi-closed set.

We have $\text{psicl}\{x\} \subseteq \Psi \text{icl}\{x\}$, $\forall x \in X$.

$\Rightarrow \cap_{x \in X} \text{psicl}\{x\} \subseteq \cap_{x \in X} \Psi \text{icl}\{x\}$. Since $\cap_{x \in X} \Psi \text{icl}\{x\} = \phi$, we have $\cap_{x \in X} \text{psicl}\{x\} = \phi$. Therefore (X, τ, \leq) is weakly ps-i- R_0 space. Hence every weakly Ψ -i- R_0 space is weakly ps-i- R_0 space.

Similarly we can prove (ii), (iii), (iv), (v), (vi), (vii).

Remark 3.3: The converse of the above theorem is not true in general it can be seen in the following example.

Example 3.4:

- (i) Let $X = \{a, b, c\}$, $\tau = \{\phi, X, \{a\}, \{a, c\}\}$ and $\leq = \{(a, a), (b, b), (c, c), (b, a)\}$ be a topological ordered space. Then (X, τ, \leq) is a weakly ps-i- R_0 space but not weakly Ψ -i- R_0 space.
- (ii) Let $X = \{a, b, c\}$, $\tau = \{\phi, X, \{a\}, \{a, c\}\}$ and $\leq = \{(a, a), (b, b), (c, c), (b, a)\}$ be a topological ordered space. Then (X, τ, \leq) is a weakly ps-i- R_0 space but not weakly i-semi- R_0 space.
- (iii) Let $X = \{a, b, c\}$, $\tau = \{\phi, X, \{a\}, \{b\}, \{a, b\}\}$ and $\leq = \{(a, a), (b, b), (c, c)\}$ be a topological ordered space. Then (X, τ, \leq) is a weakly ps-i- R_0 space but not weakly i- α - R_0 space.
- (iv) Let $X = \{a, b, c\}$, $\tau = \{\phi, X, \{a\}, \{b\}, \{a, b\}\}$ and $\leq = \{(a, a), (b, b), (c, c)\}$ be a topological ordered space. Then (X, τ, \leq) is a weakly ps-i- R_0 space but not weakly i- R_0 space.
- (v) Let $X = \{a, b, c\}$, $\tau = \{\phi, X, \{a\}, \{b\}, \{a, b\}\}$ and $\leq = \{(a, a), (b, b), (c, c)\}$ be a topological ordered space. Then (X, τ, \leq) is a weakly ps-i- R_0 space but not weakly i-pre- R_0 space.
- (vi) Let $X = \{a, b, c\}$, $\tau = \{\phi, X, \{a\}, \{b\}, \{a, b\}\}$ and $\leq = \{(a, a), (b, b), (c, c), (a, b), (a, c)\}$ be a topological ordered space. Then (X, τ, \leq) is a weakly ps-i- R_0 space but not weakly $g\alpha$ -i- R_0 space.
- (vii) Let $X = \{a, b, c\}$, $\tau = \{\phi, X, \{a\}, \{a, c\}\}$ and $\leq = \{(a, a), (b, b), (c, c)\}$ be a topological ordered space. Then (X, τ, \leq) is a weakly ps-i- R_0 space but not weakly r-i- R_0 space.

Example 3.5:

- (i) Every weakly Ψ -d- R_0 space is weakly ps-d- R_0 space.
- (ii) Every weakly d-semi- R_0 space is weakly ps-d- R_0 space.
- (iii) Every weakly d- α - R_0 space is weakly ps-d- R_0 space.
- (iv) Every weakly d- R_0 space is weakly ps-d- R_0 space.
- (v) Every weakly d-pre- R_0 space is weakly ps-d- R_0 space.
- (vi) Every weakly $g\alpha$ -d- R_0 space is weakly ps-d- R_0 space.
- (vii) Every weakly r-d- R_0 space is weakly ps-d- R_0 space.

Proof: (i)

Let (X, τ, \leq) be any weakly Ψ -d- R_0 space .

$\Rightarrow \cap_{x \in X} \Psi dcl\{x\} = \phi$. Every Ψ d-closed set is psd-closed set.

We have $psdcl\{x\} \subseteq \Psi dcl\{x\}$, $\forall x \in X$.

$\Rightarrow \cap_{x \in X} psdcl\{x\} \subseteq \cap_{x \in X} \Psi dcl\{x\}$. Since $\cap_{x \in X} \Psi dcl\{x\} = \phi$, we have $\cap_{x \in X} psdcl\{x\} = \phi$. Therefore (X, τ, \leq) is weakly ps-d- R_0 space. Hence every weakly Ψ -d- R_0 space is weakly ps-d- R_0 space.

Similarly we can prove (ii), (iii), (iv), (v), (vi), (vii).

Remark 3.6: The converse of the above theorem is not true in general it can be seen in the following example.

Example 3.7:

- (i) Let $X = \{a, b, c\}$, $\tau = \{\phi, X, \{a\}, \{a, c\}\}$ and $\leq = \{(a, a), (b, b), (c, c), (a, c), (b, c)\}$ be a topological ordered space. Then (X, τ, \leq) is a weakly ps-d- R_0 space but not weakly Ψ -d- R_0 space.
- (ii) Let $X = \{a, b, c\}$, $\tau = \{\phi, X, \{a\}, \{a, c\}\}$ and $\leq = \{(a, a), (b, b), (c, c), (a, c), (b, c)\}$ be a topological ordered space. Then (X, τ, \leq) is a weakly ps-d- R_0 space but not weakly d-semi- R_0 space.
- (iii) Let $X = \{a, b, c\}$, $\tau = \{\phi, X, \{a\}, \{b\}, \{a, b\}\}$ and $\leq = \{(a, a), (b, b), (c, c), (a, b), (c, b)\}$ be a topological ordered space. Then (X, τ, \leq) is a weakly ps-d- R_0 space but not weakly d- α - R_0 space.
- (iv) Let $X = \{a, b, c\}$, $\tau = \{\phi, X, \{a\}, \{a, c\}\}$ and $\leq = \{(a, a), (b, b), (c, c), (a, b), (c, b)\}$ be a topological ordered space. Then (X, τ, \leq) is a weakly ps-d- R_0 space but not weakly d- R_0 space.
- (v) Let $X = \{a, b, c\}$, $\tau = \{\phi, X, \{a\}, \{b\}, \{a, b\}\}$ and $\leq = \{(a, a), (b, b), (c, c), (a, c), (b, c)\}$ be a topological ordered space. Then (X, τ, \leq) is a weakly ps-d- R_0 space but not weakly d-pre- R_0 space.
- (vi) Let $X = \{a, b, c\}$, $\tau = \{\phi, X, \{a\}, \{b\}, \{a, b\}\}$ and $\leq = \{(a, a), (b, b), (c, c), (a, b), (c, b)\}$ be a topological ordered space. Then (X, τ, \leq) is a weakly ps-d- R_0 space but not weakly $g\alpha$ -d- R_0 space.
Let $X = \{a, b, c\}$, $\tau = \{\phi, X, \{a\}, \{b\}, \{a, b\}\}$ and $\leq = \{(a, a), (b, b), (c, c), (a, b), (c, b)\}$ be a topological ordered space. Then (X, τ, \leq) is a weakly ps-d- R_0 space but not weakly r-d- R_0 space.

Theorem 3.8:

- (i) Every weakly Ψ -b- R_0 space is weakly ps-b- R_0 space.
- (ii) Every weakly b-semi- R_0 space is weakly ps-b- R_0 space.
- (iii) Every weakly b- α - R_0 space is weakly ps-b- R_0 space.
- (iv) Every weakly b- R_0 space is weakly ps-b- R_0 space.
- (v) Every weakly b-pre- R_0 space is weakly ps-b- R_0 space.
- (vi) Every weakly $g\alpha$ -b- R_0 space is weakly ps-b- R_0 space.
- (vii) Every weakly r-b- R_0 space is weakly ps-b- R_0 space.

Proof: (i)

Let (X, τ, \leq) be any weakly Ψ -b- R_0 space.

$\Rightarrow \cap_{x \in X} \Psi bcl\{x\} = \phi$. Every Ψ b-closed set is psb-closed set.

We have $psbcl\{x\} \subseteq \Psi bcl\{x\}$, $\forall x \in X$.

$\Rightarrow \cap_{x \in X} psbcl\{x\} \subseteq \cap_{x \in X} \Psi bcl\{x\}$. Since $\cap_{x \in X} \Psi bcl\{x\} = \phi$, we have $\cap_{x \in X} psbcl\{x\} = \phi$. Therefore (X, τ, \leq) is weakly ps-b- R_0 space. Hence every weakly Ψ -b- R_0 space is weakly ps-b- R_0 space.

Similarly we can prove (ii), (iii), (iv), (v), (vi), (vii).

Remark 3.9: The converse of the above theorem is not true in general it can be seen in the following example.

Example 3.10:

- (i) Let $X = \{a, b, c\}$, $\tau = \{\phi, X, \{a\}, \{a, c\}\}$ and $\leq = \{(a, a), (b, b), (c, c), (a, c)\}$ be a topological ordered space. Then (X, τ, \leq) is a weakly ps-b- R_0 space but not weakly Ψ -b- R_0 space.
- (ii) Let $X = \{a, b, c\}$, $\tau = \{\phi, X, \{a\}, \{a, c\}\}$ and $\leq = \{(a, a), (b, b), (c, c), (b, a)\}$ be a topological ordered space. Then (X, τ, \leq) is a weakly ps-b- R_0 space but not weakly b-semi- R_0 space.
- (iii) Let $X = \{a, b, c\}$, $\tau = \{\phi, X, \{a\}, \{a, c\}\}$ and $\leq = \{(a, a), (b, b), (c, c), (a, c)\}$ be a topological ordered space. Then (X, τ, \leq) is a weakly ps-b- R_0 space but not weakly b- α - R_0 space.
- (iv) Let $X = \{a, b, c\}$, $\tau = \{\phi, X, \{a\}, \{b\}, \{a, b\}\}$ and $\leq = \{(a, a), (b, b), (c, c), (a, c)\}$ be a topological ordered space. Then (X, τ, \leq) is a weakly ps-b- R_0 space but not weakly b- R_0 space.
- (v) Let $X = \{a, b, c\}$, $\tau = \{\phi, X, \{a\}, \{a, c\}\}$ and $\leq = \{(a, a), (b, b), (c, c), (b, a)\}$ be a topological ordered space. Then (X, τ, \leq) is a weakly ps-b- R_0 space but not weakly b-pre- R_0 space.

- (vi) Let $X = \{a, b, c\}$, $\tau = \{\phi, X, \{a\}, \{b\}, \{a, b\}\}$ and $\leq = \{(a, a), (b, b), (c, c), (a, c)\}$ be a topological ordered space. Then (X, τ, \leq) is a weakly ps-b- R_0 space but not weakly $g\alpha$ -b- R_0 space.
- (vii) Let $X = \{a, b, c\}$, $\tau = \{\phi, X, \{a\}, \{b\}, \{a, b\}\}$ and $\leq = \{(a, a), (b, b), (c, c), (a, c)\}$ be a topological ordered space. Then (X, τ, \leq) is a weakly ps-b- R_0 space but not weakly r-b- R_0 space.

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