Fractional Order PID Controllers Applications – Some Case Studies

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Abstract: In this paper we discussed some principle definitions on fractional order integration and differentiation. The proportional- integral derivative controllers (PID) are most widely used in the application of process control due to its simple structure and easy implementation. However Fractional order PID (FOPID) controllers are generalization of the Integer order (IO) PID controllers. Here we introduced the different types of the FOPID controllers and presented some of its applications and case studies and performed a comparative study of integer and fractional order PID controllers for best outcomes.

Key words: Fractional order calculus, Fractional Order Controllers, PID controller.

I. INTRODUCTION

The concept of fractional calculus was introduced more than 300 years ago, but it was not applied in the control field until the 1960's. The Proportional-Integral-Derivative (PID) controllers are still the most widely controller in engineering and industrial for process control applications. The reason behind the popularity of PID controllers increased due to of its simple structure and easily implementation.PID controllers are still by far the most popular feedback design in industry. Fractional calculus is a generalization of differentiation and integration to a non-integer order $\alpha \epsilon C$, being the fundamental operator ${}_aD_t^{\propto}$, where a and t are the limits of the operation.The fractional order Proportional-Integral-Derivative (FOPID) was first introduced by Podlubny [1] and it consider as the generalization case of classical PID controllers. The PID controllers performance further improved by the use of fractional order derivatives and fractional order integrals.

II. BASIC CONCEPTS OF FRACTIONAL ORDER CALCULUS

The following Figure.1 shows the concept of the fractional order differ-integrations on a number line. In integer order calculus, a function f can be differentiated or integrated successively an integer number of times.

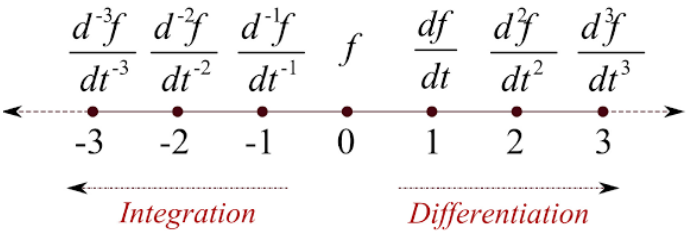

Figure.1: Number line and its interpolation for the concept of differ-integrals of fractional calculus

These are represented by the solid dotted points on the number line. However this notion of differentiation and integration can be extended to include any point on the number line which falls in-between the integer cases. This is thus a generalization of the integer order calculus and is termed as fractional order calculus.

Many applications of fractional calculus can be obtained from the area of control systems. Fractional calculus derivatives and integrals are may any real number. The fundamental operator representing the non-integer order differentiation and integration is given by ${}_aD_t^{\alpha}$, where $\alpha \in \mathbb{R}$ is the order of the differentiation or integration and 'a' and 't' are the limits of the operation. It is defined as

$$ {}_aD_t^{\alpha} = \begin{cases} \frac{d^{\propto}}{dt^{\propto}}, & \propto > 0 \\ 1, & \propto = 0 \\ \int_a^t (d\tau)^{\propto}, & \propto < 0 \end{cases} $$

The Grünwald–Letnikov (GL) and the Riemann Liouville (RL) definitions (Oldham 1974) are most commonly used definitions for the general fractional differentiation and integration:

The Grünwald–Letnikov definition is given below:

$${}_aD_t^{\alpha}f(t) = \lim_{h\to 0} h^{-\alpha} \sum_{j=0}^{\left[\frac{t-a}{h}\right]} (-1)^j \binom{\propto}{j} f(t-jh)$$ Where $\left[\frac{t-a}{h}\right]$ is an integer

While the RL definition is given by:

$${}_aD_t^{\alpha}f(t) = \frac{1}{\Gamma(n-\alpha)} \frac{d^n}{dt^n} \int_a^t \frac{f(\tau)d\tau}{(t-\tau)^{(\alpha-n+1)}} \quad (n-1 \leq \propto < n)$$

Where n is an integer and α is real a number. Γ(x) is the Gamma function. Also, there is one more definition of fractional differentiation and integration introduced by (Caputo 1967).

Caputo's definition can be written as:

$${}^c_aD_t^{\alpha}f(t) = \frac{1}{\Gamma(n-\alpha)} \int_a^t \frac{f^{(n)}(\tau)d\tau}{(t-\tau)^{(\alpha-n+1)}} \quad (n-1 \leq \propto < n)$$

Fractional order differential equations are at least as stable as their integer order counterparts.

III. THE INTEGER AND FRACTIONAL ORDER PIDCONTROLLERS

The transfer function of integer order PID controller as follows:

$$K_p + T_i S^{-1} + T_d s$$

Here, the order is unity for both integration and derivative.

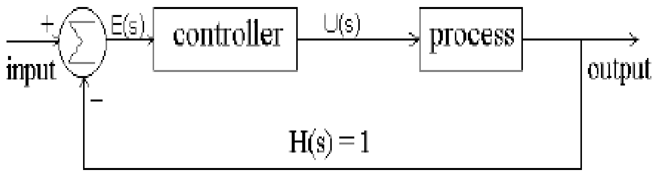

Fig. 2: Generic Closed Loop System

The processes or real objects that we want to control are usually fractional. However, for many of them the fractionality is minimum. In general, the fractional Systems with integer-order approximation can cause significant differences between real system and mathematical model. The reason behind the use of integer-order models was the non existence of solution methods for fractional-order differential equations.PID controllers belong to dominating industrial controllers and therefore are objects of steady effort for improvements of their quality and robustness. The possibility of the use fractional-order controllers with non-integer differentiation and integration improves PID controllers. A fractional PID controller therefore has the transfer function:

$$K_p + \frac{T_i}{s^{\lambda}} + T_d s^{\delta}$$

The orders of differentiation and integration are respectively δ and λ(both may not be integers, but are positive real numbers).If we take both λ and δ unity; we will obtain PID controller with integer order. Here the PID controller with integer order has three parameters, while the PID controller of fractional order has five. The generalization of integer order PID controller are the fractional order PID controller and expands integer order PID controller to from point to plane. This expansion improves more flexibility to controller design and we can control more accurately our real world processes. We will design both integer and fractional order PID controllers using the particle swarm optimization (PSO)algorithm and display the advantages of the fractional order controllers provide us where the integer order controllers fails.

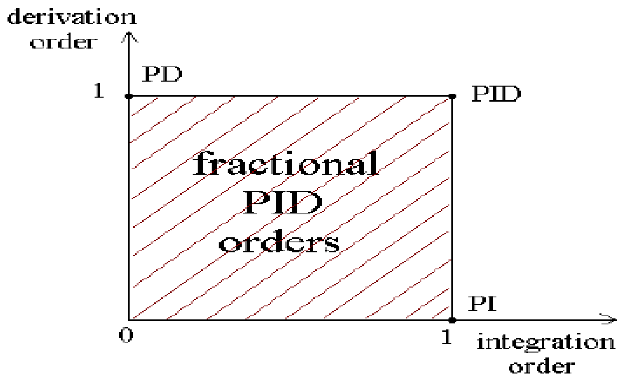

Fig. 3: Expanding from Point to Plane

1. *STABILITY OF FRACTIONAL ORDER SYSTEMS*

During the control system design the stability of fractional order system is the very fundamental and critical requirement. The known fact is that the continuous time-invariant time linear of an integer order system is stable, if and only if, characteristic polynomial has all of its roots are negative real parts. i.e, In the complex plane the roots must lie in the left half. The fractional order systems stability is the more complicated issue [5], [6].

For example, the stability of commensurate fractional order systems can by analysed via the theorem of Matington [6] or the definition from [5], which describes the way of mapping the poles from s^{α}-plane into the *w*-plane. An interesting result is that in the complex plane the poles of the stable fractional order system can be mapped even in the right half . This is shown in Fig. 4 where the stability region depicted for a commensurate fractional order linear time-invariant system with order 0<α <1 is [4], [3].

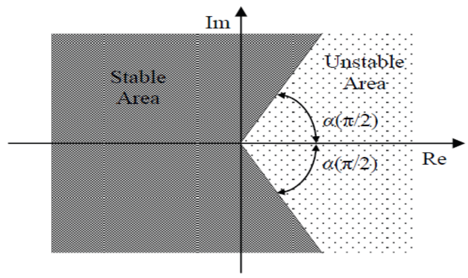

Fig. 4: Region of stability for the commensurate fractional order system with0<α <1

2. *FRACTIONAL ORDER CONTROLLERS (FOC)*

The fractional order PID (FOPID) controller is the expansion of the conventional PID controller based on fractional calculus. From many years, in industries proportional - integral - derivative (PID) controllers have been very popular in applications of process control. Their excellence consists in simplicity of design and its best performance, such as low percentage of overreach and small settling time (In slow industrial processes which are essential). The importance of PID controllers, continuous efforts are being made to improve their robustness and quality. In the automatic control field, the fractional order controllers which are the generalization integer order controllers would lead to robust control performances and more accurate. The important fact, that to attain the best performance in the fractional order models require the fractional order controllers, most of the cases the fractional order controllers are applied to regular linear dynamics or nonlinear dynamics to improve the system control performances. Historically there are four major types of fractional order controllers: (Xue and Chen, 2002)

- *CRONE Controller*
- *Tilted Proportional and Integral (TID) Controller*
- *Fractional Order PI D Controller*
- *Fractional Lead-Lag Compensator*

CRONE Controller

CRONE is a French acronym for fractional order robust control. By the use of these controllers it is possible to ensure almost constant closed loop characteristics, and ensure small variation of the closed loop system stability degree inspite of the plant perturbation and uncertainty in model parameters. It has a frequency domain design methodology employing fractional differentiation. It is possible to control minimum and non-minimum phase plants, unstable, time varying and non-linear plants with this controller. There are 3 generations of CRONE control successively extending the application fields. Some applications of these controllers have been in the domain of flexible transmission, car suspension control, hydraulic actuator (oustaloup et al. 2006.)

Tilted Proportional and Integral (TID) Controller

The purpose of TID is to provide an improved feedback loop compensator consisting the advantages of the conventional PID compensator, but provided that response which is closer to the theoretically optimal response. In TID proposal the proportional compensating unit is replaced with a compensator containing a transfer function characterized by $\frac{1}{s^{n-1}}$ or $s^{-1/n}$. This compensator is referred to as a "Tilt" compensator, as it provides a feedback gain as a function of frequency which is tilted or shaped with respect to positional compensation unit or the gain/frequency of a conventional. The entire compensator in this referred to as a Tilt-Integral-Derivative(TID) compensator. For the Tilt compensator, n is a real number not equal to zero, let it be in between 2 and 3. Thus, unlike the conventional PID controller, wherein exponent coefficients of the transfer functions of the compensator elements are either -1 or 0 or +1, TID proposal to exploits an exponent coefficient of n^{-1}. Thereplacement of the conventional proportional compensator by the tilt compensator of the invention, an overall reaction obtained which is nearer to the theoretical optimal response determined by Bode.

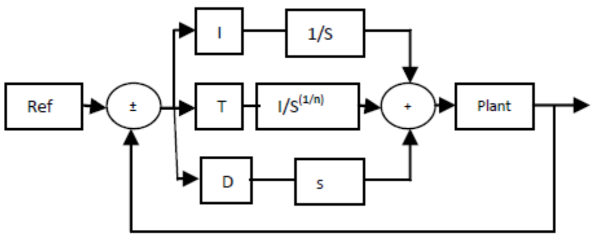

Fig. 5: Tilt-integral-derivative controller

Fractional Order PI D Controller

$PI^{\lambda}D^{\delta}$ controller was studied both in frequency domain and in time domain. In generally, the transfer function of $PI^{\lambda}D^{\delta}$ is given by

$$C(s) = \frac{U(s)}{E(s)} = k_p + \frac{k_i}{s^{\lambda}} + k_d s^{\delta}$$

Where both λ and δ are positive real numbers, k_p is the proportional gain, 'i' is the integration constant and 'd' is the differentiation constant. Clearly, takingboth λ and δ as unity, we obtain a integer order PID controller. We obtain a PD^{δ} controller,if we take $\lambda = 0$ ($k_i = 0$) etc. These all types of controllers are $PI^{\lambda}D^{\delta}$ controllers, particular cases .We can be expected that $PI^{\lambda}D^{\delta}$ controller may increase the performance of control systems due to many tuning knobs introduced. Due to the fractional order in differentiator or integrator $PI^{\lambda}D^{\delta}$ itself is an infinite dimensional linear filter.

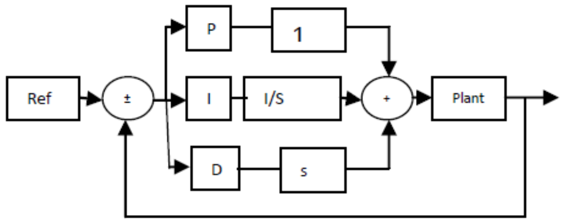

Fig.6. FO-PID ($PI^{\lambda}D^{\mu}$) controller (where, $0 \le \lambda \le 1$ & $0 \le \mu \le 1$)

Fractional Lead-Lag Compensator

Lead compensators are mostly utilised to stabilize the marginally stable systems. Lag compensators are mostly utilised to reduce the magnitude of the system high frequency loop gain. The use of fractional order elements in this lag-lead compensator gives greater flexibility to the designer, to shape the loop frequency responses since the order of the filter can take any real value instead of only integer values the transfer function of a generic FO lead-lag compensator is given by (Monje et al. 2004)

$$C(s) = K_c\left(\frac{s+\frac{1}{\lambda}}{s+\frac{1}{x\lambda}}\right)^{\alpha} = K_c x^{\alpha}\left(\frac{\lambda s+1}{x\lambda s+1}\right)^{\alpha}, 0 < x < 1$$

FOPID Controller Tuning:

The five parameters of the FOPID controller, can be used to tune the controller, thus we can achieve a higher flexibility, than in the case of an integer order PID controller. For this reason we expect to attain with the FOPID controller better closed loop performances than that the ones obtained with the integer order PID controllers. Even thoughaeasy tuning rule, as in the case of PID controllers, does not exists. For tuning FOPID controllers Barbosa [7] proposed an experimental method. i.e determining the parameters by using Ziegler-Nichols methods. The parameters of the controller are varied until system obtaining a satisfactory response.

Fractional order PID controllers applications – some case studies

Fractional calculus helps control systems hit their mark

Compared with classical (or integer-order) calculus, which forms the mathematical basis of most control systems, fractional calculus is better equipped to handle the time-dependent effects observed in real-world processes. These include the memory-like behaviour of electrical circuits and chemical reactions in batteries. By recasting the design of a set point filter as a fractional calculus problem, researchers created a filter that could not only suppress overshooting but also minimize the response time of a virtual controller.

A side-by-side comparison showed that their fractional filter outperformed an integer-order filter, tracking the complex path of a given set point more closely.

One drawback of this fractional design is that it's difficult to incorporate into existing automated systems, unlike integer-order filters, which are generally plug-and-play. But as the world of automation becomes increasingly complex, fractional filters may ultimately set the new standard for controlling everything from robotics and self-driving cars to medical devices.

Design of Fractional–order PID Controllers for Time Delay Systems using Differential Evolution Algorithm

The approach is based on a composition of the Smith predictor control method and Differential Evolution (DE) algorithm to arrive meliorated control efficiency of the time delay process. Currently, Differential Evolution (DE) has been revealed as an ordinary but very powerful thing for real parameter optimization. The five Parameters of FOPID controllers consisted, derivative constant, integral constant, derivative order and integral order, proportionality constant, thus its scheme is more complicated than that of conventional integer-order Proportional-Integral-Derivative (PID) controller. Manufacturing of the controller illustrate here is depends on user- specified peak settling time and overshoot and has been formulated as optimization issue with a single objective. Finally, better control performance and simulation results of the Fractional-Order PID (FOPID) will be obtained in these controllers in collation with those of the classical order PID controllers. For modeling FOPID controllers a smith predictor procedure for time delay systems and sagacious optimization method Proposed based on the DE algorithm. For FOPID controllers Fractional calculus can provide original and higher efficiency extension. even though, the difficulties of calculating FOPID controllers addition, because FOPID controllers also take into account the integral order and derivative order in comparison with common PID controllers. Using fractional PID controller we have significantly reduce percentage over rise time and settling time. Simulation results gives that the fractional PID controller has better-quality performance than integer PID controller.

IV. CONCLUSION

In this paper we observed that FOPID controllers has five parameters and provided two additional degrees (powers λ and μ) in transfer function to tune the controller, where as IOPID controllers having only three and FOPID controllers shows a considerable improvement in the performance to achieve a higher flexibility when comparing with IOPID controllers.

So we conclude that from the applications and some case – studies, that FOPID controller can set the new standard for controlling everything comparing with IOPID controllers.

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