Effects of Nonlinear Convection and Variable Properties on Darcy Flow of Non-Newtonian Fluid over a Rotating Cone

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Abstract: In this paper, we investigated unsteady non-linear convective flow of a Casson fluid from a rotating cone in Darcy saturated porous medium. Flow through porous medium is described by Darcy’s law. Viscosity and thermal conductivity are assumed to be varied and depends on temperature. The governing equations of the pertinent model are non-dimensionalized and reduced into non-linear coupled ordinary differential equations. The resultant equations are solved numerically and the results are presented graphically for physical parameter, non-linear thermal and solutal convection parameters, variable viscosity and variable thermal conductivity parameters on flow velocity, temperature and concentration profiles. The obtained results are validated by comparing with earlier results.

Keywords: Variable viscosity, Thermal conductivity, Non-linear convection, Non-Newtonian, Numerical Technique.

I. INTRODUCTION

In the orthodox handling of temperature boundary, habitually viscosity and conductivity are presumed to be persistent. Nevertheless the investigational inquiry evoke that this belief is trustworthy only if the temperature alteration during the movement is not too vast. For oiling fluids, warmth is spawned by the intramural abrasion and the analogous upswing in temperature alters the thickness of fluid. Thus the fluid thickness and conductivity are no longer presumed constants. For paradigm the conclusive viscosity and conductivity of water diminutions by 240% when the temperature escalations from 100°C to 500°C. Hence in order to foresee the flow behaviour suspiciously, it is imperative and indispensable to consider the temperature dependent viscosity and conductivity. In the view of this Pinarbasi et al. [1] observed the impact of dependent thermal conductivity and thickness for non-isothermal fluid flow. Sivaraj & Kumar [2] analysed the electric conductivity variation with fate plate and moving vertical cone on viscoelastic fluid flow. Adegbie et al. [3] examined the variation of variable viscosity on micro polar fluid flow at stagnation point with melting temperature exchange. Hayat et al. [4] investigated the variable conductivity and viscosity in the mixed convective flow with the unsteadiness. Malik et al. [5] reviewed the dissipative viscous fluid flow through spinning cone with mixed convection and variable viscosity and conductivity. Palani et al. [6] considered natural convection variations with thermal conductivity and variable viscosity on a vertical cone. Ram et al. [7] attempted magnetic nanofluid flow with time dependency through a stretched spinning plate and properties of variable viscosity. Salawu and Dada [8] researched radioactive temperature change of variable conductivity and viscosity in a non-Darcy medium on inclined magnetic field with dissipation. Umavathi et al. [9] used viscous fluid flow with the properties of variable viscosity and conductivity with first order chemical reaction and mixed convection in a vertical channel. Konch and Hazarika [10] studied Dusty fluid hydromagnetic flow through a spinning vertical cone with the effects of thermal conductivity and variable viscosity. Kumar et al. [11] discussed the time dependent clutched flow of tangent hyperbolic fluid with variable conductivity through a sensor surface. Chenlin et al. [12] depicted the nonlinear transient effects of diffusivity and variable conductivity with universal diffusion-thermo elasticity and time domain finite element analysis.

More over with the fluid movement, it is also essential to elect the accurate dominion of the movement which is pertinent to the real world requests. The heat transmission and mass transmission through a spinning cone is significant for the model of innumerable engineering apparatus like heat swaps, containers for fissionable trash, discarding and geothermal basins. Anilkumar and Roy [13] depicted spinning fluid flow with mixed convection and
unsteadiness through a revolving cone. Roy et al. [14] discussed time dependent MHD flow of circling fluid over a gyration cone. Chamkha and Rashad [15] studied Soret mixed convection by unsteady mass and heat transmission on fluid flow over a turning cone. Nadeem and Saleem [16] researched about a nanofluid through a rotating cone with magnetic field and time dependent mixed convection. Saleem and Nadeem [17] reviewed theoretical analysis of viscous rakishness changes over a spinning cone with slip flow. Mallikarjuna et al. [18] attempted MHD convective mass and heat transfer with chemical reaction effect over a revolving cone fixed in an inconstant porosity regime. Rashad et al. [19] considered thermophoresis and thermal radiation effects in a porous medium over a turning cone with mass and heat transfer. Saleem et al. [20] examined chemical reaction and heat generation on second order viscoelastic time dependent fluid flow past a gyration cone. Sulochana et al. [21] investigated numerically the Brownian motion and thermophoresis over a revolving cone. Dinarvand and Pop [23] considered the viscosity and thermal conductivity as a function of temperature. We consider the rectangular curvilinear fixed coordinate system. The geometry of the problem is shown in Fig. 1. Let x, y and z be the velocity components along the tangential, circumferential and normal directions respectively. The cone is in a rigid body rotation about the axis of a cone with timed dependent angular velocity. The rotation of the cone causes the unsteadiness in the fluid flow. The temperature and concentration variations are responsible for the buoyancy forces in the fluid. The surface of cone is an electrically-insulated. The magnetic Reynolds number is taken to be small such that the effect of induced magnetic field is negligible. Further, the electric field is absent in the flow. According to above assumptions the boundary layer equations of momentum, energy and diffusion for an incompressible, unsteady flow are given by Malik et al. [26]

The governing equations are

\[
X \frac{\partial U}{\partial X} + U \frac{\partial X}{\partial Z} = 0 \tag{1}
\]

\[
\frac{\partial U}{\partial t} + U \frac{\partial U}{\partial X} + W \frac{\partial U}{\partial Z} = \frac{1}{\rho} \left( \frac{1}{\rho} \frac{\partial}{\partial X} \left( \mu \frac{\partial U}{\partial X} \right) - \frac{\mu}{\rho} U + \frac{\mu}{\rho} V \right) \tag{2}
\]

\[
\frac{\partial V}{\partial t} + U \frac{\partial V}{\partial X} + W \frac{\partial V}{\partial Z} = \frac{1}{\rho} \left( \frac{1}{\rho} \frac{\partial}{\partial X} \left( \mu \frac{\partial V}{\partial X} \right) - \frac{\mu}{\rho} V \right) \tag{3}
\]

\[
\frac{\partial T}{\partial t} + U \frac{\partial T}{\partial X} + W \frac{\partial T}{\partial Z} = \frac{1}{\rho c_p} \left( \alpha \frac{\partial T}{\partial Z} \right) \tag{4}
\]

\[
\frac{\partial C}{\partial t} + U \frac{\partial C}{\partial X} + W \frac{\partial C}{\partial Z} = D \frac{\partial^2 C}{\partial Z^2} \tag{5}
\]

The boundary conditions are

\[
U = 0, V = 0, W = 0, T = T_s, C = C_s \text{ at } Z = 0 \tag{6}
\]

and

\[
U = 0, V = 0, T = T_s, C = C_s \text{ as } Z \to \infty
\]

Where U, V and W are velocities along X, Y and Z directions respectively.

III. SOLUTIONS AND PROCEDURE

Introducing the following similarity transformations and non-dimensional variables

\[
U = \frac{1}{2} \left( \Omega \sin \alpha \right) \left( 1 - \frac{\eta}{L} \right) f(\eta), \quad V = \frac{1}{2} \left( \Omega \sin \alpha \right) \left( 1 - \frac{\eta}{L} \right) g(\eta), \quad W = \frac{1}{2} \left( \Omega \sin \alpha \right) \left( 1 - \frac{\eta}{L} \right) h(\eta),
\]

\[
T = T_s + (T_c - T_s) \theta(\eta), \quad C = C_s + (C_c - C_s) \phi(\eta)
\]

\[
(7)
\]

The physical model of the problem is shown in Fig. 1.
The converted governing equations are

\[
\left( 1 + \frac{1}{\beta} \right) \frac{\Theta}{\Theta} f' + \left( 1 + \frac{\Theta}{\Theta} \right) f'' - s \left( f' + \frac{1}{2} \varphi' \right) - \varphi' + \frac{1}{2} (f')^2 \right) - D \left( 1 + \frac{\Theta}{\Theta} \right) f - 2 \frac{\Theta}{\Theta} \varphi - 2 \Delta \left( \Theta + \alpha \Theta' + N \left( \Theta + \alpha \Theta' \right) \right) = 0
\]

(8)

\[
\left( 1 + \frac{1}{\beta} \right) \frac{\Theta}{\Theta} g' + \left( 1 + \frac{\Theta}{\Theta} \right) g'' - s \left( g' + \frac{1}{2} \varphi' \right) - \varphi' = 0
\]

(9)

g' - Du \left( 1 + \frac{\Theta}{\Theta} \right) g = 0

And the associated boundary conditions are

\[
f' = 0, g = 1, f = 0, \Theta = 1, \Phi = 1 \text{ at } \eta = 0
\]

and \( f' = 0, g = 0, \Theta = 0, \Phi = 0 \text{ as } \eta \to \infty \)

**Method of Solution**

The nonlinear ordinary differential equations (ODEs) (8)-(11) with the boundary limitations (12) have been numerically solved using Shooting technique with the help of Runge-Kutta method. Initially, the set of nonlinear ODEs converted to 1" order differential equations, by using the following process:

\[
f = f_1, f' = f_2, f'' = f_3, g = f_4, g' = f_5, \Theta = f_6, \Phi = f_7 \text{ and } \Theta = f_8
\]

Then the above equations will be converted to the following form

\[
f'' = \left( 1 + \frac{1}{\beta} \right) \frac{\Theta}{\Theta} \left[ s f_1 + 2 \frac{\Theta}{\Theta} f_2 + f_1 f_2 - \frac{1}{2} f_1^2 \right] - D \left( 1 + \frac{\Theta}{\Theta} \right) f_1 + 2 \Theta f_2 + 2 \Delta \left( f_1 + \alpha \Theta f_2 + N \left( f_1 + \alpha \Theta f_2 \right) \right)
\]

(10)

\[
g'' = \left( 1 + \frac{1}{\beta} \right) \frac{\Theta}{\Theta} \left[ s f_3 + 2 \frac{\Theta}{\Theta} f_4 + f_1 f_2 - \frac{1}{2} f_1^2 \right]
\]

(11)

The corresponding boundary conditions for mass transfer analysis are given by

\[
f_1 = 0, f_2 = 0, f_3 = 1, f_4 = 1, f_5 = 1 \text{ at } \eta = 0
\]

\[
f_2 \to 0, f_3 \to 0, f_4 \to 0, f_5 \to 0 \text{ as } \eta \to \infty
\]

We guess the values of \( f_4(0), f_5(0), g_4(0) \) which are not given at the initial conditions. The equations (8)-(11) are integrated by taking the help of Runge-Kutta method, with the successive iterative step length is 0.01. The correctness of the supposed values is verified by equating the calculated values \( f_2, f_4, f_6, f_8 \) at \( \zeta = \zeta_{\text{max}} \) with their given values at \( \zeta = \zeta^{\text{max}} \). If there is any difference exist the process is continued up to the required good values. Alternatively, we are using the Runge-Kutta method to get the accurately found the initial values of \( y_2, y_6, y_8 \) and then integrated (8)-(11) by using the Runge-Kutta method. This process is continued until the completion between the designed value and the condition given at is within the specified degree of accuracy \( 10^{-5} \).

**IV. RESULTS AND DISCUSSIONS**

The Analytical solutions of a set of equations (8) – (10) with conditions (12) are difficult due to coupled non-linearity. Therefore these equations are solved numerically using Runge-Kutta based shooting method. Results shows the influence of non-dimensional governing parameters on velocity and temperature profiles along with the friction factor coefficient and local Nusselt and Sherwood numbers for three cases (Linear convection, opposing and assisting flows). For numerical calculations we measured the non-dimensional parameter values as \( \beta = 0.5, S = 2, \Delta = 10, \Pr = 2, Sc = 0.22, R = 0.5, \Gamma = 0.25, N = 1, \theta = 0.2 \).

These values are reserved as constant in whole study excluding the dissimilarities in the particular figures. The obtained results are compared and correlated with existing results Hering and Grosh [27] for linear convection of a Newtonian fluid without porous media as shown in Table-1.

<p>| Table-1: Comparison of Results |
|-----------------------------|-----------------------------|</p>
<table>
<thead>
<tr>
<th>( \Delta )</th>
<th>( -\Theta'(0) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.022</td>
<td>0.42852</td>
</tr>
<tr>
<td>0.17</td>
<td>0.46156</td>
</tr>
<tr>
<td>1.0</td>
<td>0.61207</td>
</tr>
<tr>
<td>10</td>
<td>1.0173</td>
</tr>
</tbody>
</table>

The effects of nonlinear thermal and concentration convection parameters (\( \alpha_1 \) and \( \alpha_2 \)) on velocities (\( f(\eta), f'(\eta) \) and \( g(\eta) \)), temperature and concentration profiles (\( \Theta(\eta) \) and \( \Phi(\eta) \)) are plotted in Figs. 2-6. The enlightening values of \( \alpha_1 \) and \( \alpha_2 \) diminish the normal, circumferential velocities, temperature and concentration profiles, whereas the mixed behaviour (increment and decrement) in tangential velocity. The raising values of the nonlinear convection in the flow improve the fluid particle interaction nonlinearily; this leads to reduction in temperature and concentration and mixed behaviour in...
tangential velocity.

Figs. 2-11 display the graphical demonstration of velocities (normal, azimuthal and tangential directions), temperature and concentration profiles for various values of Casson fluid and unsteadiness parameters. The promotions in $\beta$ and $s$ initiated shrink in normal, azimuthal velocities, temperature and concentration profiles and mixed nature (increment and decrement) in the tangential velocity profile. For increasing values of Casson fluid improves the boundary layer viscosity nature of the flow due to this we saw decrement in velocities. Similarly, unsteady parameter encourages unsteadiness in the flow.
The effects of viscosity variation and conductivity variation parameters (\(\phi\) and \(\Gamma\)) on \(f' (\eta)\), \(g (\eta)\), \(\theta (\eta)\) and \(\phi (\eta)\) are plotted in Figs. 12-15. It is evident that the \(g(\eta), \theta(\eta)\) and \(\phi(\eta)\) profiles are declined as improving values of viscous variation and conductivity variation parameters, whereas the mixed performance (increase and decrease) was showed in \(f'(\eta)\). As rising values of viscosity and conductivity variation improves the variation between the fluid particles, due to this we saw decrement in \(g(\eta), \theta(\eta)\) and \(\phi(\eta)\).
Figs. 16-19 are plotted to show the variations of frictions between the particles in $x$ and $y$ directions ($C_f_x$ and $C_f_y$), Nusselt and Sherwood numbers ($Nu$ and $Sh$) on nonlinear thermal and concentration convection parameters ($\alpha_1$ and $\alpha_2$). The friction between the particles in $x$ and $y$ directions ($C_f_x$ and $C_f_y$), Nusselt and Sherwood numbers ($Nu$ and $Sh$) are amended with growing values of nonlinear thermal and concentration convection parameters ($\alpha_1$ and $\alpha_2$).

The effects of Casson fluid and unsteadiness parameters ($\beta$ and $s$) on frictions in $x$ and $y$ directions, Nusselt and Sherwood numbers are intriguing in Figs. 20-23. A rise in Casson fluid and unsteadiness parameters initiated enlargement in friction between the particles in $x$ and $y$ directions, Nusselt and Sherwood numbers. Since, as improved values of Casson and unsteady improves the thickness of the boundary and unsteadiness in the flow.
Figs. 24-27 are plotted to show the variations of frictions between the particles in x and y directions (Cf_x and Cf_y), Nusselt and Sherwood numbers (Nu and Sh) on viscosity variation and conductivity variation parameters (\( \Theta \) and \( \Gamma \)). The friction between the particles in x and y directions (Cf_x and Cf_y), Nusselt and Sherwood numbers (Nu and Sh) are amended with growing values of viscosity and conductivity variation parameters (\( \Theta \) and \( \Gamma \)).

**V. CONCLUSIONS**

This paper addresses the flow characteristics of Casson fluid flow on a spinning cone with porous medium. Viscosity and thermal conductivity are assumed to be varied and depends on temperature. The governing equations of the pertinent model are non-dimensioned and reduced into non-linear coupled ordinary differential equations. The resultant equations are solved numerically and the results are presented graphically for physical parameter, non-linear thermal and solutal convection.
parameters, variable viscosity and variable thermal conductivity parameters on flow velocity, temperature and concentration profiles. The obtained results are validated by comparing with earlier results. The conclusions of the present study are as follows.

- The friction between the particles in x and y directions is more with the increasing values of nonlinear thermal and concentration convection parameters.
- The increasing values of Casson fluid and unsteadiness parameters improve the heat and mass transfer rates in the flow.
- The viscosity and conductivity variation improves the friction between the particle and heat transfer rate.
- The non-Newtonian Casson fluid parameter depreciates the momentum and thermal boundary layers.

**REFERENCES**


[18] Mallikarjuna, B., Rashad, A. M., Chamkha, A. J., & Raju, S. H. Chemical reaction effects on MHD


