2(1) - Prime Partial Ideals Of Partial Γ- Semirings

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Abstract: A partial Γ-semiring is a structure possessing an infinitary partial addition and a ternary multiplication, satisfying a set of identities. The partial functions under disjoint-domain sums and functional composition is a partial Γ-semiring. In this paper we introduce the notions of 2(1)-prime partial ideal and m2(m1)-system and obtained some relations between them in partial Γ-semirings.

Keywords: prime ideal, m-system, 2(1)-prime partial ideal, m2(m1)-system of (R, I), 2(1)-spec(R) & 2(1)-prime radical of K.

I. INTRODUCTION


In this paper we introduce the notions of 2(1)-prime partial ideal and m2(m1)-system in a partial Γ-semiring and obtain the characterizations of intersection of all 2(1)-prime partial ideals of R and 2(1)-prime radical of K.

II. PRELIMINARIES

In this section we collect some important definitions and results for our use in this paper.

1.1. Definition. [5] A partial monoid is a pair (M, Σ) where M is a nonempty set and Σ is a partial addition defined on some, but not necessarily all families (x_i : i ∈ I) in M subject to the following axioms:

(i) Unary Sum Axiom. If (x_i : i ∈ I) is a one element family in M and I = { j }, then Σ(x_i : i ∈ I) is defined and equals x_j.

(ii) Partition-Associativity Axiom. If (x_i : i ∈ I) is a family in M and (I_j : j ∈ J) is a partition of I, then (x_i : i ∈ I) is summable if and only if (x_i : i ∈ I_j) is summable for every j ∈ J, (Σ(x_i : i ∈ I) : j ∈ J) is summable, and (Σ(x_i : i ∈ I) = Σ{Σ(x_i : i ∈ I_j) : j ∈ J}).

1.2. Definition. [8] Let (R, ⊔) and (Γ, ⊖) be two partial monoids. Then R is said to be a partial Γ-semiring if there exists a mapping R × Γ × R → R (images to be denoted by xγy for x, y ∈ R and γ ∈ Γ) satisfying the following axioms:

(i) xγ(yμz) = (xγy)μz,
1.7. Definition. [10] Let R be a partial $\Gamma$-semiring. If $A, B$ are subsets of $R$ and $\Gamma_1$ is a subset of $\Gamma$, define $A \Gamma_1 B$ as the set $\{x \in R \mid \exists a, \alpha \in A, \gamma, \beta \in \Gamma_1, b_i \in B, \Sigma a \alpha \beta_i, \text{exists and } x = \Sigma a \alpha \beta_i \}$.

If $A = \{a\}$ then we also denote $A \Gamma_1 B$ by $a \Gamma_1 B$. If $B = \{b\}$ then we also denote $A \Gamma_1 B$ by $A \Gamma_1 b$. Similarly if $A = \{a\}$ and $B = \{b\}$, we denote $A \Gamma_1 B$ by $a \Gamma_1 b$ and thus $a \Gamma_1 b = \{x \in R \mid x = a \gamma b \text{ for some } \gamma \in \Gamma_1\}$.

An ideal A of a $\Gamma$-so-ring R is called proper if A $\neq$ R.

1.8. Definition. [12] A proper partial ideal P of a partial $\Gamma$-semiring R is said to be prime if and only if for any partial ideals A, B of R, $A \cap B \subseteq P$ implies $A \subseteq P$ or $B \subseteq P$.

2. 2(1)-PRIME PARTIAL IDEALS

We introduce the notion of 2(1)-prime partial ideals in partial $\Gamma$-semirings as follows:

2.1. Definition. Let R be a partial $\Gamma$-semiring and A be a proper partial ideal of $(R, \Gamma)$. Then A is said to be 2-prime if and only if for any subtractive partial ideals I, J of $(R, \Gamma)$, $I J \subseteq A$ implies $I \subseteq A$ or $J \subseteq A$.

2.2. Definition. Let R be a partial $\Gamma$-semiring and A be a proper partial ideal of $(R, \Gamma)$. Then A is said to be 2-prime if and only if for any subtractive partial ideal I and a partial ideal J of $(R, \Gamma)$, $I J \subseteq A$ implies $I \subseteq A$ or $J \subseteq A$.

2.3. Remark. Let R be a partial $\Gamma$-semiring and A be a proper partial ideal of $(R, \Gamma)$. If A is a prime ideal of $(R, \Gamma)$ then A is a 2(1)-prime partial ideal of $(R, \Gamma)$.

**Proof.** Suppose A is a prime partial ideal of $(R, \Gamma)$. Let I, J be any two subtractive partial ideals of $(R, \Gamma)$ such that $I J \subseteq A$. Then I, J are partial ideals of $(R, \Gamma)$ such that $I J \subseteq A$. Since A is prime, $I \subseteq A$ or $J \subseteq A$. Hence A is a 2(1)-prime partial ideal of $(R, \Gamma)$.

The following example illustrates that in a partial $\Gamma$-semiring R, a 2(1)-prime partial ideal of $(R, \Gamma)$ is not prime.

2.4. Example. Let $R = \{0, 1, 2, 3\}$. Define $\Sigma$ on R as

$\Sigma = \{x_j, \text{ if } x_j = 0 \text{ for some } j \}$.

Then $(R, \Sigma)$ is a partial monoid. Let $\Gamma := N \cup \{0\}$, the set of all non-negative integers. Then $\Gamma$ is a partial monoid with finite support addition. Define a mapping $R \times \Gamma \times R$ into $R$ as

$$
\begin{align*}
0 & \text{ if any one of } x, \alpha, y \text{ is } 0, \\
x & \text{ if } \alpha = y = 1, \\
(x, \alpha, y) & = \alpha \text{ if } x = y = 1, \quad \alpha \leq 3, \\
y & \text{ if } x = \alpha = 1, \\
3 & \text{ if } x \alpha, y \geq 3
\end{align*}
$$

Then R is a partial $\Gamma$-semiring. The partial ideals of $(R, \Gamma)$ are $[0], \{0, 2\}, \{0, 3\}, \{0, 2, 3\}, R$ and the subtractive partial ideals of $(R, \Gamma)$ are $\{0\}$ and R. Since $\{0, 2, 3\} \subseteq \{0, 2, 3\} = A$ and $\{0, 2, 3\} \subseteq I$, 1 is not a prime partial ideal of $(R, \Gamma)$. But it easy to prove that I is a 2(1)-prime partial ideal of $(R, \Gamma)$.

2.5. Lemma. Let R be a complete partial $\Gamma$-semiring and A be a partial ideal of $(R, \Gamma)$. Then the subtractive closure of A is $\overline{A} = \{a \in R \mid a + x \in A, \text{ for some } x \in A\}$.

**Proof.** Take $S = \{a \in R \mid a + x \in A, \text{ for some } x \in A\}$. First we prove that S is the subtractive partial ideal of $(R, \Gamma)$ containing A. Let $(a_i : i \in I)$ be a (summable) family in R such that $a_i \in S \forall i \in I$. Then $\forall i \in I, a_i + x \in A$ for some $x_i \in A$. Now $\Sigma a_i = \Sigma x_i = \Sigma (a_i + x_i) \in A$ and hence $\Sigma a_i \in S$. Let $r \in R, \alpha \in \Gamma, r \in S$. Then $r \in R, \alpha \in \Gamma$ and $a + x \in A$ for some $x \in A$. Now $r \alpha(a + x) = r \alpha a + r \alpha x \in A$ and $(a + x) \alpha = a \alpha + x \alpha a \in A$ and hence $r \alpha a, a \alpha \in S$. Thus S is a partial ideal of $(R, \Gamma$). Let $a, b \in R$ be such that $a + b \in S$ and $b \in S$. Then $a + b + x \in A$ and $b + y \in A$ for some $x, y \in A$. Thus $a + b + x + y \in A$ for some $b + x + y \in A$ and hence $a \in S$. Thus S is a subtractive partial ideal of $(R, \Gamma)$. Since $a + 0 = a \in A, A \subseteq S$. Now we prove that S is the smallest subtractive partial ideal of $(R, \Gamma)$ containing A. Let $P$ be another subtractive partial ideal of $(R, \Gamma)$ containing A. Let $a \in S$. Then $a + x \in A$ for some
Let \( x \in A \). Since \( A \subseteq P, a + x \in P \) and \( x \in P \). Since \( P \) is subtractive, \( a \in P \) and hence \( A \subseteq P \). Hence the lemma.

2.6. Theorem. Let \( A \) be a partial ideal of a partial \( \Gamma \)-semiring \( R \). Then \( A \) is \( 2(1) \)-prime partial ideal if and only if

\[
\begin{align*}
&\text{such that} \\
&\langle x \rangle \Gamma \langle y \rangle \subseteq A \langle x \rangle \Gamma \langle y \rangle \subseteq A \text{ then } x \in A \text{ or } y \in A.
\end{align*}
\]

Proof. Suppose \( A \) is a \( 2(1) \)-prime partial ideal of \( (R, \Gamma) \). Let \( x, y \in R \) such that \( \langle x \rangle \Gamma \langle y \rangle \subseteq A \). Since \( \langle x \rangle \Gamma \langle y \rangle \) are partial ideals of \( (R, \Gamma) \) such that \( \langle x \rangle \Gamma \langle y \rangle \subseteq A \) and \( A \) is \( 2(1) \)-prime, \( \langle x \rangle \subseteq A \) or \( \langle y \rangle \subseteq A \). Hence \( x \in A \) or \( y \in A \).

Conversely, suppose that \( x, y \in R \) such that \( \langle x \rangle \Gamma \langle y \rangle \subseteq A \) then \( x \in A \) or \( y \in A \). Let \( X, Y \) be any two partial ideals of \( (R, \Gamma) \) such that \( XY \subseteq A \). Suppose \( X \not\subseteq A \). Then \( \exists x \in X \) \( \forall x \not\in A \).

Let \( y \in Y \). Now we prove that \( \langle x \rangle \Gamma \langle y \rangle \subseteq XY \).

Let \( a \in \langle x \rangle \Gamma \langle y \rangle \). Then \( a = \sum_{i} x_{i}a_{i}y_{i} \) for some \( x_{i} \in \langle x \rangle, a_{i} \in \Gamma, y_{i} \in \langle y \rangle \) \( \forall i \in I \). Since \( x_{i} + x_{i}' \in \langle x \rangle \subseteq X \) for some \( x_{i}' \in \langle x \rangle \subseteq X \) \( \forall i \in I \). Similarly we prove that \( y_{i} \in Y \) \( \forall i \in I \). Thus \( a = \sum_{i} x_{i}a_{i}y_{i} \subseteq XY \).

Hence \( \langle x \rangle \Gamma \langle y \rangle \subseteq XY \). By assumption, \( x \in X \) or \( y \in Y \). Since \( x \not\in X, y \not\in Y \) and hence \( Y \not\subseteq A \). Hence the theorem.

2.7. Definition. If \( X, Y \) are non-empty subsets of a partial \( \Gamma \)-semiring \( R \), then

(i) \( (X:Y)_{t} = \{a \in R \mid a \Gamma Y \subseteq X\} \)

(ii) \( (X:Y)_{s} = \{a \in R \mid Y \Gamma a \subseteq X\} \)

(iii) \( (X:Y) = \{a \in R \mid a \Gamma Y \subseteq X \text{ and } Y \Gamma a \subseteq X\} \).

2.8. Lemma. Let \( R \) be a partial \( \Gamma \)-semiring. Then

(i) If \( X \) and \( Y \) are left(right) partial ideals of \( (R, \Gamma) \) then \( (X:Y)_{t}((X:Y)_{s} \) is a partial ideal of \( (R, \Gamma) \).

(ii) If \( X \) is a subtractive left(right) partial ideal and \( Y \) is a left(right) partial ideal of \( (R, \Gamma) \) then \( (X:Y)_{t}((X:Y)_{s} \) is a subtractive partial ideal of \( (R, \Gamma) \).

Proof. (i): we have \( (X:Y)_{t} = \{a \in R \mid a \Gamma Y \subseteq X\} \). Let \( (a_{i}:i \in I) \) be a summable family in \( R \) and \( a_{i} \in (X:Y)_{t} \). Then \( x_{i} \Gamma B \subseteq A \) \( \forall i \in I \).

\[
\begin{align*}
&\sum_{i} a_{i} \Gamma Y \subseteq X. \Rightarrow \sum_{i} a_{i} \Gamma Y \subseteq X. \Rightarrow \sum_{i} x_{i} \in (X:Y)_{t}.
\end{align*}
\]

Let \( r \in R, \alpha \in \Gamma, a \in (X:Y)_{t} \). Then \( r \in R, \alpha \in \Gamma \) and \( a \Gamma Y \subseteq X \). Consider \( (r \alpha a) \Gamma Y = r \alpha (a \Gamma Y) \). Since \( a \Gamma Y \subseteq X, r \alpha (a \Gamma Y) \subseteq r \alpha X \). Since \( X \) is a left partial ideal, \( r \alpha X \subseteq X. \Rightarrow (r \alpha a) \Gamma Y \subseteq X. \Rightarrow r \alpha a \in (X:Y)_{t} \).

Consider \( (a \alpha r) \Gamma Y = a \alpha (r \Gamma Y) \). Since \( Y \) is a left partial ideal of \( (R, \Gamma), r \Gamma Y \subseteq Y. \Rightarrow a \alpha (r \Gamma Y) \subseteq a \alpha Y \subseteq a \Gamma Y \subseteq X \).

\( \Rightarrow (a \alpha r) \Gamma Y \subseteq X. \Rightarrow a \alpha r \in (X:Y)_{t} \). Therefore \( (X:Y)_{t} \) is a subtractive partial ideal of \( (R, \Gamma) \).

(ii): Suppose \( X \) is a subtractive left ideal and \( Y \) is a left partial ideal of \( (R, \Gamma) \).

Let \( a, a+b \in (X:Y)_{t} \). \( \Rightarrow a \Gamma Y \subseteq X \) and \( (a+b) \Gamma Y \subseteq X \). Let \( c \in b \Gamma Y \). Then \( c = \sum_{i} b_{i} \alpha_{i} y_{i} \) for some \( \alpha_{i} \in \Gamma, y_{i} \in Y \). Now \( a \alpha_{i} y_{i} \in a \Gamma Y \) \( \forall i \in I \).

\( \Rightarrow x \alpha_{i} b_{i} \in A \) \( \forall i \in I \). \( \Rightarrow x \alpha_{i} y_{i} \in a \Gamma Y \). Now \( \sum_{i} a \alpha_{i} y_{i} + \sum_{i} b_{i} \alpha_{i} y_{i} = \sum_{i} (a+b) \alpha_{i} y_{i} \subseteq (a+b) \Gamma Y \subseteq X \).

Since \( X \) is subtractive, \( \sum_{i} a \alpha_{i} y_{i} \subseteq X \) and \( \sum_{i} b_{i} \alpha_{i} y_{i} + c \subseteq X \). Hence \( b \Gamma Y \subseteq X. \Rightarrow b \in (X:Y)_{t} \). Therefore \( (X:Y)_{t} \) is a subtractive partial ideal of \( (R, \Gamma) \).

2.9. Theorem. Let \( A \) be a subtractive partial ideal of a partial \( \Gamma \)-semiring \( R \). Then \( A \) is prime if and only if \( A \) is \( 2(1) \)-prime.

Proof. By Remark 2.3., if \( A \) is prime then \( A \) is \( 2(1) \)-prime.

Conversely, suppose that \( A \) is \( 2(1) \)-prime partial ideal of \( (R, \Gamma) \). Let \( X, Y \) be any two partial ideals of \( (R, \Gamma) \) \( \exists X \subseteq (A:Y)_{t} \). Since \( A \) is subtractive, by lemma 2.8.(ii), \( (A:Y)_{t} \) is a subtractive partial ideal containing \( X \).

\( \Rightarrow \overline{X} \subseteq (A:Y)_{t} \). \( \Rightarrow \overline{X} \Gamma Y \subseteq A \). \( \Rightarrow Y \subseteq (A: \overline{X}) \).

Since \( A \) is subtractive, by lemma 2.8.(ii), \( (A: \overline{X})_{t} \) is a subtractive partial ideal containing \( Y \).

\( \Rightarrow \overline{Y} \subseteq (A: \overline{X})_{t} \). \( \Rightarrow \overline{X} \Gamma \overline{Y} \subseteq A \). Since \( A \) is \( 2(1) \)-prime and \( \overline{X}, \overline{Y} \) are subtractive partial ideals \( \overline{X} \subseteq A \) or \( \overline{Y} \subseteq A \).

Thus \( \overline{X} \subseteq A \) or \( \overline{Y} \subseteq A \). Hence \( A \) is a prime partial ideal of \( (R, \Gamma) \).
3. $m_2(m_1) - \text{SYSTEM OF } (R, \Gamma)$

We define $m_2(m_1)$-system in a partial $\Gamma$-semiring as follows:

**3.1. Definition.** Let $R$ be a partial $\Gamma$-semiring and $A$ be a subset of $R$. Then $A$ is called an $m_2(m_1)$-system of $(R, \Gamma)$ if for any $a, b \in A$, $a < a', b \in < b >$, $r \in R$, $a, b \in \Gamma$ such that $a_i a_i b_i \in A$. Hence $A$ is an $m_2(m_1)$-system of $(R, \Gamma)$.

The following example illustrates that an $m_2(m_1)$-system is not m-system in a partial $\Gamma$-semiring $R$.

**3.3. Example.** Consider the partial $\Gamma$-semiring $R$ as in the example 2.4. Then a subset $A = \{1, 2\}$ of $R$ is an $m_2(m_1)$-system. For $2 \in A$, for any $r \in R$ and $\alpha, \beta \in \Gamma$, $2\alpha\beta = 3 \notin A$. Hence $A = \{1, 2\}$ is not an $m$-system of $(R, \Gamma)$.

**3.4. Lemma.** Let $R$ be a partial $\Gamma$-semiring with left/right unity and $P$ be a proper partial ideal of $(R, \Gamma)$. Then $P$ is a 2(1)-prime partial ideal of $(R, \Gamma)$ if and only if $R \setminus P$ is an $m_2(m_1)$-system of $R$.

**Proof.** Suppose $P$ is a 2(1)-prime partial ideal of $(R, \Gamma)$. Let $a, b \in R \setminus P$. Then $a, b \notin P$. By theorem 2.6.,

$$a > b > a \notin P \Rightarrow \exists \quad x \in < a > \cap < b > \ni x \notin P \ni x = \sum_{i} a_i b_i \quad \text{for some}$$

$$a_i \in \langle a \rangle, b_i \in \langle b \rangle.$$ Since

$$x \notin P, \sum_{i} a_i b_i \notin P. \Rightarrow \exists \quad a_i' \in \langle a \rangle, a_i' \in \langle b \rangle \ni a_i' a_i' b_i' \in P \Rightarrow a_i' a_i' b_i' \notin P.$$

Hence $R \setminus P$ is an $m_2(m_1)$-system of $R$.

Conversely suppose that $R \setminus P$ is an $m_2(m_1)$-system of $R$. Let $a, b \in R \setminus P \ni a' \sim < a > \ni b \in < b > \ni P$. Suppose $a \notin P$ and $b \notin P$. Then $a, b \in R \setminus P$. Suppose $a \notin P$ and $b \notin P$. Then $a, b \in R \setminus P$. Suppose $a \notin P$ and $b \notin P$. Then $a, b \in R \setminus P$. Suppose $a \notin P$ and $b \notin P$. Then $a, b \in R \setminus P$. Suppose $a \notin P$ and $b \notin P$. Then $a, b \in R \setminus P$. Suppose $a \notin P$ and $b \notin P$. Then $a, b \in R \setminus P$. Suppose $a \notin P$ and $b \notin P$. Then $a, b \in R \setminus P$. Suppose $a \notin P$ and $b \notin P$. Then $a, b \in R \setminus P$. Suppose $a \notin P$ and $b \notin P$. Then $a, b \in R \setminus P$. Suppose $a \notin P$ and $b \notin P$. Then $a, b \in R \setminus P$. Suppose $a \notin P$ and $b \notin P$. Then $a, b \in R \setminus P$. Suppose $a \notin P$ and $b \notin P$. Then $a, b \in R \setminus P$. Suppose $a \notin P$ and $b \notin P$. Then $a, b \in R \setminus P$. Suppose $a \notin P$ and $b \notin P$. Then $a, b \in R \setminus P$. Suppose $a \notin P$ and $b \notin P$. Then $a, b \in R \setminus P$. Suppose $a \notin P$ and $b \notin P$. Then $a, b \in R \setminus P$. Suppose $a \notin P$ and $b \notin P$. Then $a, b \in R \setminus P$. Suppose $a \notin P$ and $b \notin P$. Then $a, b \in R \setminus P$. Suppose $a \notin P$ and $b \notin P$. Then $a, b \in R \setminus P$. Suppose $a \notin P$ and $b \notin P$. Then $a, b \in R \setminus P$. Suppose $a \notin P$ and $b \notin P$. Then $a, b \in R \setminus P$. Suppose $a \notin P$ and $b \notin P$. Then $a, b \in R \setminus P$. Suppose $a \notin P$ and $b \notin P$. Then $a, b \in R \setminus P$. Suppose $a \notin P$ and $b \notin P. Then
Suppose \((A : X)_r \cap M \neq \phi \Rightarrow \exists x \in (A : X)_r \cap M \neq \phi \). Then \(x \in (A : X)_r \cap M \neq \phi \). Since \(x \in M \). Hence \(\exists x \leq (A : X)_r \Rightarrow (A : X)_r \subseteq A \). Thus \(\forall x \in M \). Hence \(\forall x \in (A : X)_r \). Thus \(\forall x \in (A : X)_r \subseteq (A : X)\).

Case(i): If \(X \cap M = \phi \). Then \(\exists a \in (X \cap M = \phi) \Rightarrow a \in (X \cap M = \phi) \). Then \(\exists a \in (X \cap M = \phi) \Rightarrow a \in (X \cap M = \phi) \). Since \(\exists a \in (X \cap M = \phi) \Rightarrow a \in (X \cap M = \phi) \). Therefore \(A = (A : X) \). Hence \(A = (A : X) \). Since \(\forall x \in (X \cap M = \phi) \). Hence \(\forall x \in (A : X) \). Hence \(\forall x \in (A : X) \). Hence \(\forall x \in (A : X) \).

Case(ii): If \(X \cap M = \phi \). Since \(\forall x \in (X \cap M = \phi) \). Hence \(\forall x \in (A : X) \). Hence \(\forall x \in (A : X) \). Hence \(\forall x \in (A : X) \).

3.6. Theorem. Let \(R\) be a partial \(\Gamma\)-semiring. Then every 2(1)-prime partial ideal \(I\) of \((R, \Gamma)\) contains a minimal 2(1)-prime partial ideal of \((R, \Gamma)\).

Proof. Take \(\tau = \{A | A \text{ is a 2(1)-prime ideal of } (R, \Gamma) \} \). Clearly \(I \in \tau\). Hence \((\tau, \subseteq)\) is a non-empty poset. Let \(\{B_j | j \in J\}\) be a descending chain of elements in \(\tau\). Take \(B := \bigcap_{j \in J} B_j\). Then \(B\) is clearly a partial ideal of \((R, \Gamma)\) which contains \(I\). Now let \(x, y \in R \Rightarrow x \geq y \Rightarrow x \in B\). Then \(x \notin B_k\) for some \(k \in J\). If \(j \leq k\). Since \(\exists x \geq y \Rightarrow y \notin B_k\) and \(x \notin B_k\). \(x \in B_j\) and \(y \in P_{j}\). Since \(j \leq k\). Then \(B_j \subseteq B_k\). Since \(x \notin B_j\). \(x \notin B_j\). Now \(\exists x \geq y \Rightarrow x \in B_j\) and \(x \notin B_j\). \(x \in B_j\). \(x \in B_j\) and \(y \in B_j\) \(\forall j \in J\). Hence \(y \in B_j\) for all \(j \in J\).\( \Rightarrow y \in \bigcap_{j \in J} B_j = B\). Hence \(B\) is a 2(1)-prime partial ideal of \((R, \Gamma)\) which contains \(I\). Thus \(B\) is a minimal of \(\{B_j | j \in J\}\) in \(\tau\). By Zorn’s lemma, \(\tau\) has a minimal element.

The set of all 2(1)-prime partial ideals of \((R, \Gamma)\) is denoted by \(2(1)-\spec(R)\).

3.7. Theorem. Let \(R\) be a partial \(\Gamma\)-semiring with left/right unity. Then \(\bigcap (2(1)-\spec(R)) = \{x \in R | \text{there is an m}_2(x)\text{-system A of R with } x \in A \text{ implies } 0 \in A\} \).

Proof. Take \(T := \{x \in R | \text{there is an m}_2\text{-system A of R with } x \in A \text{ implies } 0 \in A\} \). Let \(x \notin \bigcap 2(1)-\spec(R)\). Then \(x \notin P\) for some 2(1)-prime partial ideal \(P\) of \((R, \Gamma)\). Since \(P\) is a 2(1)-prime partial ideal of \((R, \Gamma)\), \(R \setminus P\) is an \(m_2\text{-system of } (R, \Gamma)\). Since \(0 \notin P\), \(0 \notin R \setminus P\). Thus \(\exists a \in m_2\text{-system } R \setminus P\). \(\exists x \in R \setminus P\) and \(0 \notin R \setminus P\). \(\Rightarrow \exists x \notin T\). Therefore \(\exists x \notin \bigcap 2(1)-\spec(R)\). Let \(x \notin T\). Then \(\exists a \in m_2\text{-system A of } R \Rightarrow x \notin A\). Now \(\exists x \in A\) is a 2(1)-prime partial ideal of \((R, \Gamma)\) \(\Rightarrow \exists x \notin \bigcap 2(1)-\spec(R)\). Therefore \(\exists x \notin \bigcap 2(1)-\spec(R)\). Hence \(\bigcap 2(1)-\spec(R)\). □

3.8. Definition. Let \(K\) be a proper partial ideal of a partial \(\Gamma\)-semiring \(R\) with left/right unity. Then the 2(1)-prime radical of \(K\) is defined as the smallest 2(1)-prime partial ideals which contains \(K\) and is denoted by \(2(1)-\sqrt{K}\). i.e., \(2(1)-\sqrt{K} = \{x \in R | \text{there is an m}_2\text{-system A of R with } x \in A \text{ implies } A \cap \sqrt{K} = \phi\} \).

3.9. Theorem. If \(K\) is a proper partial ideal of a partial \(\Gamma\)-semiring \(R\) then \(2(1)-\sqrt{K} = \{x \in R | \text{there is an m}_2\text{-system A of R with } x \in A \text{ implies } A \cap \sqrt{K} = \phi\} \). Let \(x \notin T\). Then \(\exists a \in m_2\text{-system A of } R \Rightarrow x \notin A\). Now \(\exists a \in m_2\text{-system A of } R \Rightarrow x \notin A\) and \(A \cap \sqrt{K} = \phi\). By theorem 3.5., \(\exists a \in m_2\text{-system A of } R \Rightarrow x \notin A\). Hence \(A \cap \sqrt{K} = \phi\) and \(P\) is maximal with respect to this property.

Since \(x \in A\), \(x \notin P\). Since \(P \subseteq \sqrt{K}, x \notin P\) for some 2(1)-prime partial ideal \(P\) of \((R, \Gamma)\). \(\Rightarrow x \notin \bigcap 2(1)-\spec(R)\). Therefore \(\bigcap 2(1)-\spec(R)\). Hence \(\bigcap 2(1)-\spec(R)\). □

IV. CONCLUSION

In this paper we introduced the notions of 2(1)-prime partial ideal and \(m_2(x)\)-system in a partial \(\Gamma\)-semiring and proved that \(P\) is a 2(1)-prime partial ideal of \(R\) if and only if \(R \setminus P\) is an \(m_2(x)\)-system of \((R, T)\). Also we obtained the
characterizations of intersection of all 2(1)-prime partial ideals of $R$ and 2(1)-prime radical of $K$.

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